TRIGONOMETRY

WITH APPLICATIONS

THIRD EDITION

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Appendix A

Graphing—Addition of Ordinates

Although the advent of desktop computers and calculators with graphics capabilities has given many users of trigonometry the ability to graph a function without putting pencil to paper, there are often instances where it is useful to be able to obtain a quick sketch of a function by hand. The method we examine in this section, **addition of ordinates,** can be very helpful in this regard. Also, studying this method leads to a fuller understanding of functions, and this understanding will aid even the person using a computer's graphing capabilities. We assume a thorough knowledge of section 3–3.

We learned in section 2–1 that a function is a set of ordered pairs in which no first element repeats. When we graph a function in a rectangular coordinate system, we use two perpendicular axes. We use horizontal distances to represent domain elements, or values of the argument of the function. Vertical distances are used to represent range elements, or values of the function for a given value of the argument. In this way, each ordered pair represents a domain element and the corresponding range element. In an ordered pair, the first element is sometimes called the **abscissa;** the second element is then called the **ordinate.** For example, consider the graph of a function f, shown in figure A–1. Although we do not know the ''formula'' for f (i.e., an expression that defines f), or even whether such a formula exists, we can still see that $f(1) = 2$ and $f(2) = 4$. We know that $f(1) = 2$, because if we go a horizontal distance of 1, corresponding to domain element 1, or an abscissa of 1, we see that the associated vertical distance is 2, representing a range element, or ordinate, of 2.

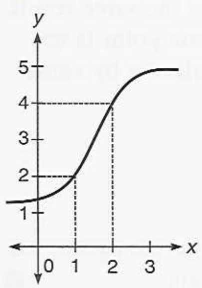


Figure A–1

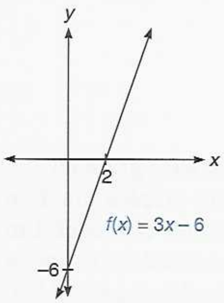
In general, if we go a horizontal distance x, the corresponding vertical distance represents f(x).

Since we use the y-axis to plot values of $f(x)$, we often write $y = f(x)$, or replace $f(x)$ by y in a formula.

The graph of a linear function is relatively simple. A linear function is a function of the form $f(x) = mx + b$. (We often simply write $y = mx + b$.) The graph of such a function is a straight line. This means that if we can find two points on the line (i.e., that satisfy the function) we know the graph is a straight line which must pass through them.

Thus, to graph a linear function, we find two points that lie on the line and draw a straight line through them. Usually these two points are the x- and y-intercepts.

Example A



1. Graph the linear function $f(x) = 3x - 6$.

   It is easier to do the necessary algebra if we replace $f(x)$ by y, giving $y = 3x - 6$.

   To find the x-intercept we replace y by 0, since for any point on the x-axis, y is 0.

$$y = 3x - 6$$
$$0 = 3x - 6$$
$$2 = x$$

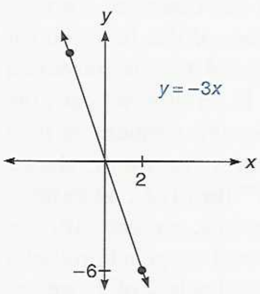
   Therefore, the x-intercept is (2,0).

   To find the y-intercept we replace x by 0.

$$y = 3x - 6$$
$$y = 3(0) - 6$$
$$y = -6$$

   Therefore, the y-intercept is $(0,-6)$.

   In the figure we plot these two points and draw the straight line through them.



2. Graph the linear function $f(x) = -3x$.

   First we write $y = -3x$. Replacing x by 0 or y by 0 gives the same result, (0,0). The origin is, therefore, the only intercept. Since one point is not enough to determine a straight line, we find another. Replace x by some value other than 0, say 2.

$$y = -3x$$
$$y = -3(2)$$
$$y = -6$$

   Therefore, a second point that lies on the line is $(2,-6)$. In the figure, we plot both points and draw the line that passes through them.

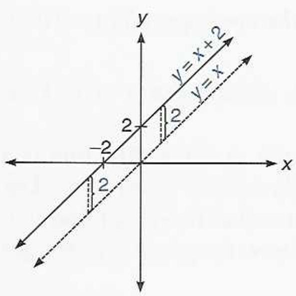


Figure A–2

Now, consider the graph of $f(x) = x + 2$. First, we shall relabel this as $y = x + 2$. Its graph is shown in figure A–2. If we graph the equation $y = x$, shown in dashed lines in figure A–2, we see that every point in the graph of $y = x + 2$ is two units above every point in the graph of $y = x$. Another way to view this is that the graph of $y = x + 2$ is the sum of the graphs of $y = x$ and $y = 2$. This summing is done vertically. (We are adding the ordinates.) That is, to graph $y = x + 2$ for a given value of x, we take the vertical distance in the graph of $y = x$ and add the vertical distance in $y = 2$. This process is shown for $x = 3$ and for $x = -5$ in figure A–3. Note that we treat each distance as a directed distance. If the function is positive, we move up; if negative, we move down.

For example, at the point $x = 3$, we move up three units and then two more units. The length 3 represents the value of the equation $y = x$ when x is 3. The length 2 represents the value of the equation $y = 2$ when x is 3.

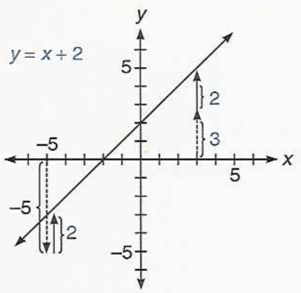


Figure A–3

At the point $x = -5$, we add vertical lengths -5 and 2. The length -5 (5 in the downward direction) represents the value of the equation $y = x$ when x is -5, and the length 2 represents the value of the equation $y = 2$ when x is -5.

Figure A–4 illustrates the process of addition of ordinates in general. We assume that we have a function h described by an expression that is the sum of the expressions for two functions f and g. That is, $h(x) = f(x) + g(x)$. For the purpose of illustration, $f(x)$ might be the expression $0.25 + \sin x$, and $g(x)$ might be the expression $\cos(x + 0.5)$. Then, $h(x)$ would be

$$\begin{aligned} h(x) &= f(x) + g(x) \\ &= (0.25 + \sin x) + [\cos (x + 0.5)] \\ &= \sin x + \cos(x + 0.5) + 0.25 \end{aligned}$$

In figure A–4 we see the graphs of the two functions, f and g, and the addition of ordinates at x_1, x_2, x_3, x_4, and x_5.

Figure A–4(a) shows the process at x_1. $f(x_1)$ and $g(x_1)$ are both positive, so the value of $h(x_1)$ is represented by the combined heights of f and g at this point.

Figure A–4(b) shows the case where $f(x_2) = g(x_2)$. Since these values are equal, the combined value $f(x_2) + g(x_2)$ is twice the height of either f or g at this point.

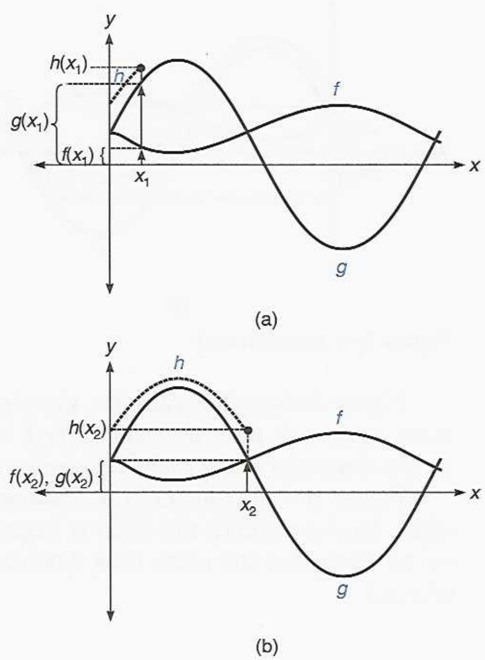
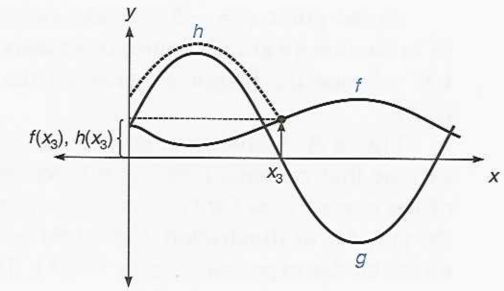
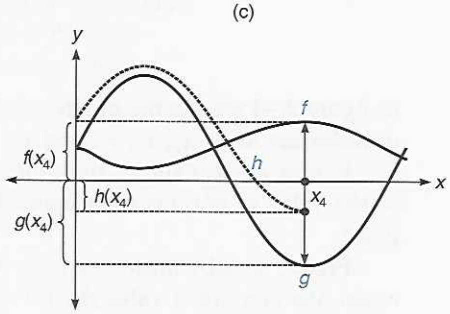


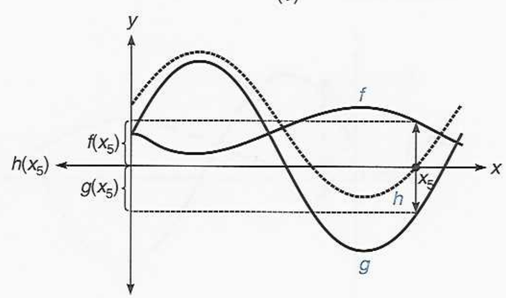
Figure A–4



(c)



(d)



(e)

Figure A–4 *(continued)*

Figure A–4(c) illustrates the situation where one function is 0 at a point. Here, $g(x_3) = 0$; thus, the sum of $f(x_3) + g(x_3)$ is just $f(x_3)$. In other words, at x_3, the functions h and f take on equal values.

Figure A–4(d) illustrates a situation where the functions have opposite signs. $f(x_4)$ is positive and $g(x_4)$ is negative. Thus, to form the value of $h(x_4)$ we go up to $f(x_4)$ and come back down a distance corresponding to the height of $g(x_4)$.

Figure A–4(e) is like the situation at x_4, except that $f(x_5)$ and $g(x_5)$ have equal absolute values. Thus, their sum is 0, so that $h(x_5) = 0$.

This discussion can provide *guidelines for finding the graph of a sum of two functions using addition of ordinates.*

1. Wherever the two functions cross, the result is twice the height of the functions (as in figure A–4 (b)).
2. Wherever one function is zero the result is the same as the other function (figure A–4 (c)).
3. Wherever the functions have opposite signs but equal absolute values the result is 0 (figure A–4 (e)).

Using these guidelines we can often obtain quite a few points in the result. We must then graph enough intermediate points (as in figure A–4 (a) and (d)) to get a good idea of what the result looks like.

The function $f(x) = x + \sin x$ provides a concrete example. First we rewrite $y = x + \sin x$ for convenience. We view this as the sum of the graphs of $y = x$ and $y = \sin x$. See figure A–5, where the graph of each of these functions is shown.

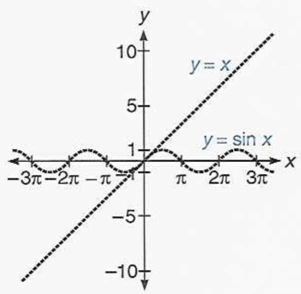


Figure A–5

To obtain the graph of $y = x + \sin x$, we proceed in the following way. We select an abscissa (moving a given horizontal distance along the x-axis) and then add the ordinate of $y = x$ and $y = \sin x$. This is shown for several abscissas in figure A–6. Observe that wherever the function $y = \sin x$ is zero, the result is the other function, $y = x$. This is guideline 2, above. Many other points in the graph of $y = x + \sin x$ are also shown in figure A–6, on the dotted line. We graph only enough points to see what the finished graph looks like. Actually, with a little experience, we often require only a few points to see the result. The graph of $y = x + \sin x$ is shown in figure A–7. We can view this as the function $y = \sin x$ ''riding on'' the function $y = x$.

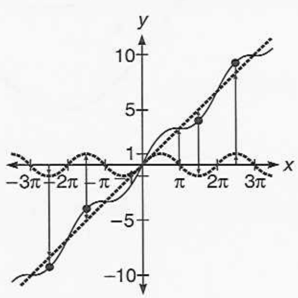


Figure A–6

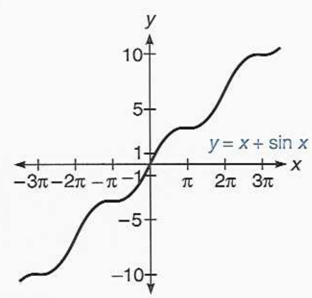
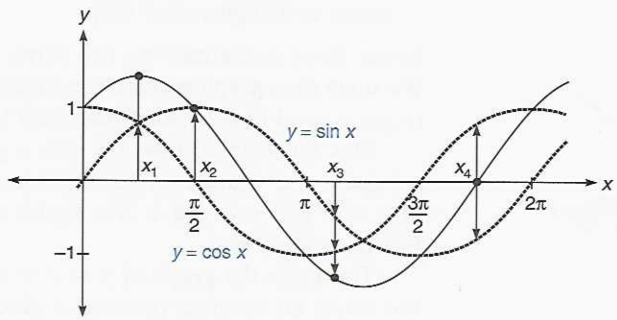


Figure A–7

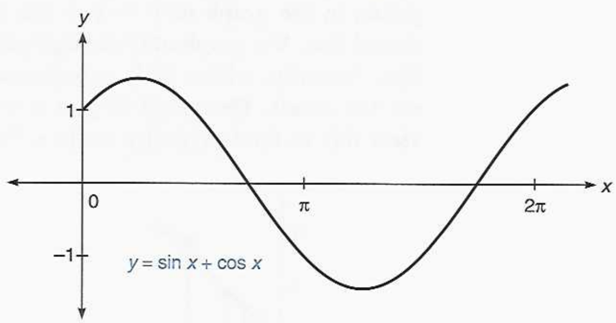
■ *Example B*

1. Graph the equation $y = \sin x + \cos x$ for $0 \le x \le 2\pi$.

The figure shows the graphs of $y = \sin x$ and $y = \cos x$ in dashed lines, along with the additions of ordinates for several values of x. In particular, at x_1 the functions have equal values and so the result is twice the height of either. At x_2 one function is zero, so the result is the same as the nonzero function. At x_3 both functions are negative, but there is nothing special about either (i.e., none of the three guidelines apply). We must add the signed distances here. At x_4 the functions have equal absolute values but opposite signs. Thus, the result is 0 at this point.



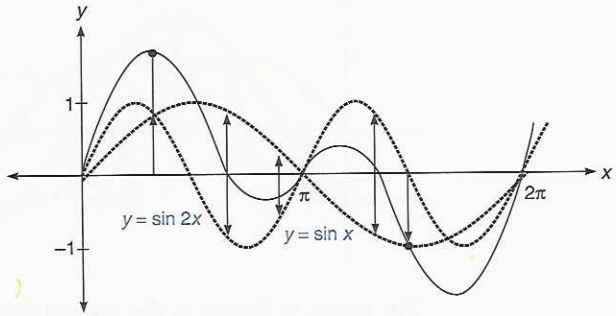
The result is shown at many other points also, on the solid line. The graph of $y = \sin x + \cos x$ is shown in the next figure.



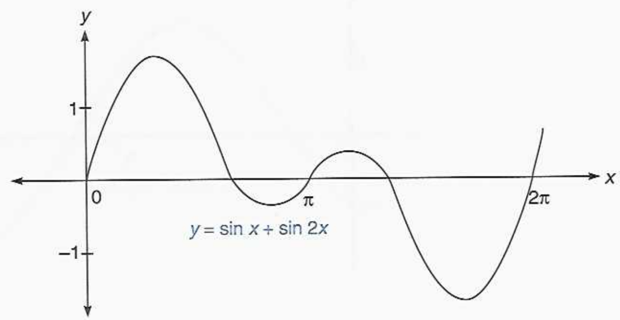
Note It can be shown that $\sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$.

2. Graph the equation $y = \sin x + \sin 2x$ for $0 \leq x \leq 2\pi$.

The figure shows the graphs of $y = \sin x$ and $y = \sin 2x$ in dashed lines, along with the additions of ordinates for several values of x, and the result at many other points (solid line).



The result is shown in the next figure.



If the expression includes subtraction, we rewrite it in terms of addition.

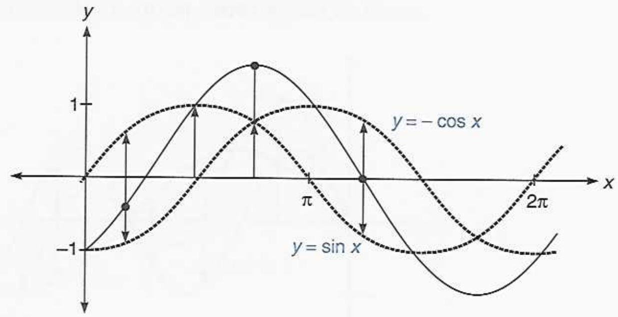
■ ***Example C***

Graph the equation $y = \sin x - \cos x$ for $0 \leq x \leq 2\pi$.

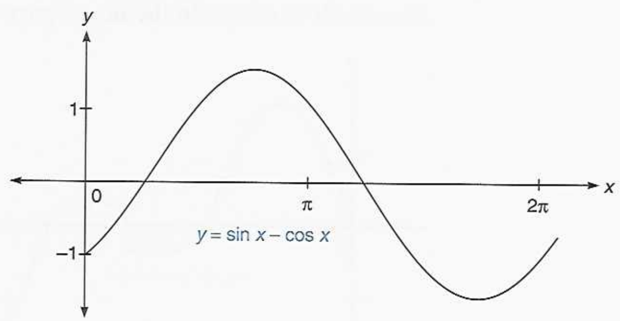
Since it is easier to picture addition of ordinates than to picture subtraction, we rewrite the equation as

$$y = \sin x + (-\cos x)$$

We now graph $y = \sin x$ and $y = -\cos x$. These are shown in the first figure with the additions of ordinates for several values of x and the result on the solid line.



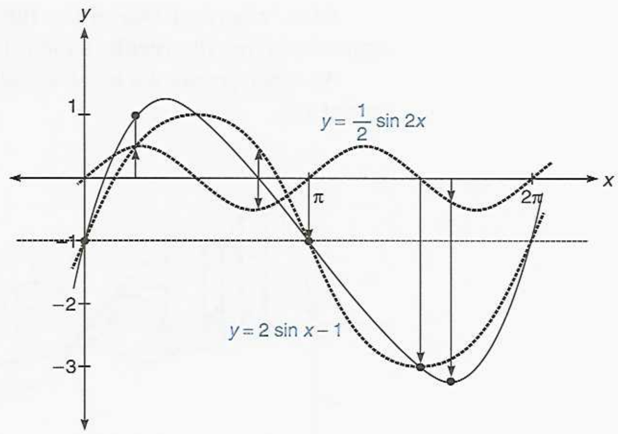
The result is shown in the second figure.



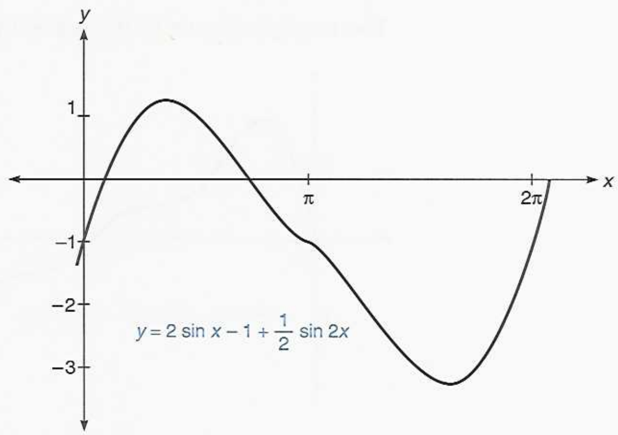
Note It can be shown that $\sin x - \cos x = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$.

Example D

1. An electronic signal that is described by the equation $y = 2 \sin x - 1$ is serving as a carrier for another signal described by $y = \frac{1}{2} \sin 2x$. That is, the finished signal is described by $y = 2 \sin x - 1 + \frac{1}{2} \sin 2x$. Graph this signal for $0 \le x \le 2\pi$.
   We will graph $y = 2 \sin x - 1$ and $y = \frac{1}{2} \sin 2x$ in dashed lines. Although we could graph $y = 2 \sin x - 1$ itself by the method of addition of ordinates, we should recall that this is just the graph of $y = 2 \sin x$ shifted down by one unit (section 3–3). The graphs of these two signal components are shown in the first figure, as well as the addition of several ordinates. The results are shown for many points on the solid line.



The result is shown in the second figure.



2. Assume a pure musical tone is being represented by $y = \sin x$. Its second harmonic is then $y = \frac{1}{2}\sin 2x$, and $y = \frac{1}{3}\sin 3x$ represents the third harmonic. Assume that all three tones are present and describe the result graphically.

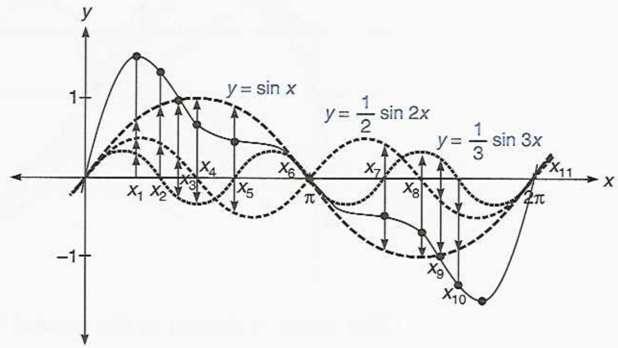
We graph the three functions separately and then add the ordinates of all three. The three graphs are shown in the first figure, as are the addition of ordinates for several values of x and the result for many points on the dotted line.

There are some points that can be of help in a complicated case like this.

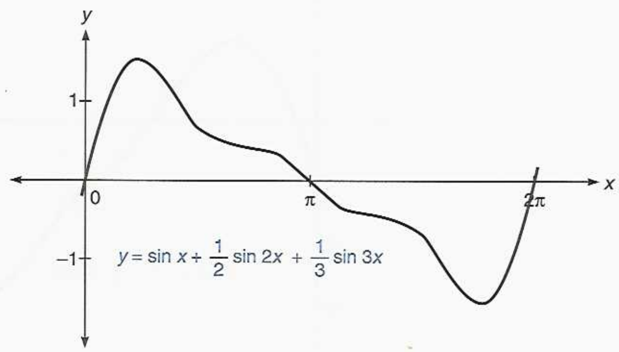
Wherever one of the functions is zero, the result is the sum of the other two functions. Thus, good selections for x are values where any of the three functions cross the x-axis (points x_2, x_4, x_5, x_6, x_7, x_8, x_{10}, and x_{11}).

Also, wherever two of the functions have equal absolute values but opposite signs, the result is the third function (points x_3 and x_9).

At other points we must simply compute the sum for three values (point x_1).



The result is shown in the second figure.



Mastery points

Can you

- Graph a function using addition of ordinates?

Exercise A

Graph the following linear functions.

1. $f(x) = 5x - 3$

2. $f(x) = -2x + 7$

3. $f(x) = -x + 3$

4. $f(x) = \frac{1}{2}x + 2$

$\boxed{\textbf{5.}}$ $f(x) = \frac{2}{3}x - 1$

$\boxed{\textbf{6.}}$ $f(x) = -2x$

Graph the following functions, using addition of ordinates, for $0 \le x \le 2\pi$.

7. $y = x + 2 \sin x$
8. $y = -x + \sin x$
9. $y = 2x + 2 \sin x$
10. $y = 2x + \sin 2x$
11. $y = \sin x + \sin 3x$
12. $y = \sin 2x + \cos x$
13. $y = \sin 2x + \cos 3x$
14. $y = 2x - \sin x$
15. $y = -x - \sin x$
16. $y = 2x - 2 \sin x$
17. $y = \sin x - \sin 3x$
18. $y = \sin x - \cos x$
19. $y = \sin 2x - \cos x$

20. The equation of time combines the effects of the inclination of the earth in its orbit about the sun, and the eccentricity of the orbit. It gives the difference between mean solar time and actual solar time throughout the year. The function that describes the effect of the eccentricity of the earth's orbit can be approximately described by $y = 7.5 \sin \frac{\pi}{6}x$, and the function that describes the effect of the inclination of the earth is approximately $y = 10 \sin \frac{\pi}{3}x$. The amplitudes tell the number of degrees of deviation between mean and actual solar time.

To find the combined effects, we graph the sum of the two functions, $y = 7.5 \sin \frac{\pi}{6}x + 10 \sin \frac{\pi}{3}x$. Graph this equation of time for $0 \le x \le 12$ (for 12 months).

21. The electric voltage supplied to United States homes has a frequency of 60 cycles per second (cps), or a period of $\frac{1}{60}$ of one second. It can be described by the function $y = 180 \sin 120\pi x$, where the amplitude is instantaneous voltage and x is in seconds.

a. Graph this function. Use divisions of $\frac{1}{240}$ of one second, and graph two complete cycles.

b. Suppose that a wind-driven home generator is generating electricity, but due to a fault in its speed governor it is generating at 50 cps and is only producing 25 volts. Assuming that these sources are in phase at some point and that they are tied together (electrically speaking), the resulting voltage can be described by $y = 180 \sin 120\pi x + 25 \sin 100\pi x$ (until a fuse blows). Graph this resultant voltage for $0 \le x \le \frac{1}{25}$.

22. Most people have heard about the theory of biorhythms. First stated by Dr. Wilhelm Fliess in Germany at the beginning of the century, this theory maintains that at birth three cycles are started—physical, emotional, and intellectual. These have periods of 23, 28, and 33 days, respectively, and can be (presumably) described by sinusoidal waves. With a time axis in days, we compute the equation for the physical as follows:

$$0 \le x \le 23$$
$$\frac{2\pi}{23}(0) \le \frac{2\pi}{23}x \le \frac{2\pi}{23}(23)$$
$$0 \le \frac{2\pi}{23}x \le 2\pi$$

Thus, the equation for the physical state is $y = \sin \frac{2\pi}{23}x$.

a. Find the equations for the emotional and intellectual cycles.

b. Graph the function that is the sum of these three cycles using addition of ordinates. Graph it for the first 33 days of life. (All three functions start together, or are in phase, at birth.)

c. Find the period of the resulting function, in both days and years.

Appendix B

Further Uses for Graphing Calculators and Computers

The use of a scientific calculator has been assumed throughout this text. We have also shown the use of graphing calculators. This appendix presents problems that are best done *only* with a graphing calculator.

Specific examples refer to the Texas Instruments *TI-81* programmable, graphing calculator and a Casio *fx*-7000G programmable, graphing calculator. It is assumed that the user has read at least the introductory material in the handbook for the calculator.

Finding zeros of functions

This topic was introduced in section 5–4. We revisit it here with examples that are difficult to solve algebraically. We also show how to accomplish these same tasks with the Casio and Texas Instruments calculators.

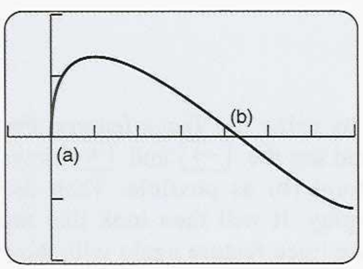
Estimating zeros from the graph

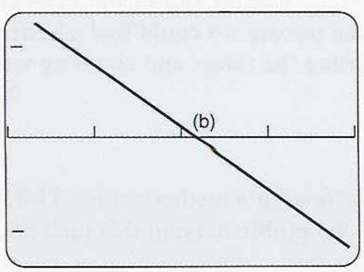
The approximate zeros of a function can be found by graphing the function. This is because a zero of a function is an x-intercept of the function's graph.

■ ***Example A***

Find approximate zeros to the function $f(x) = 2\sin(\sqrt{2x}) - \frac{1}{2}$ for $0 \le x < 2\pi$ by graphing.

Texas Instruments TI-81 Calculator





The following steps would produce a graph similar to that shown in the upper screen in the figure. It is essential that the [MODE] key be used to set the calculator to radian (Rad) mode. As can be seen in the graph the function has

Steps	Explanation
[Range]	Enter the x- and y-axis limits.
0 [ENTER]	Xmin becomes 0.
7.5 [ENTER]	Xmax becomes 7.5.
1 [ENTER]	Xscl becomes 1.
[(−)] 2.5 [ENTER]	Ymin becomes −2.5.
2.5 [ENTER]	Ymax becomes 2.5.
1 [ENTER]	Yscl becomes 1.
[2nd] [CLEAR]	Quit

Enter the function into Y_1.

[Y=] 2 [sin] [(] [2nd] [x^2] [(] 2 [XT] [)] [)] [−] 0.5 [GRAPH]

zeros at approximate x-values of 0.1 and 4.1. It would be possible to obtain a better estimate by using the trace feature. For example, select $\boxed{\text{TRACE}}$ and use the $\boxed{\Rightarrow}$ and $\boxed{\Leftarrow}$ keys to move the blinking dot as close to the point (b) as possible. Then use $\boxed{\text{ZOOM}}$ 2 $\boxed{\text{ENTER}}$ to expand the display. It will then look like the lower screen. Using the $\boxed{\text{TRACE}}$ feature again will show that point (b) is between the values 4.16 and 4.20. The trace feature shows the current value of x and y in the display. By noting the values of x when the value of y changes sign we can find estimates of the value of x at the zero at (b). By repeating the zoom and trace features we could find a better and better estimate of the zero at (b). By resetting the range and zooming we could repeat the process for the zero at (a).

Casio *fx*-7000G Calculator

The following steps will produce a graph similar to that shown in the previous figure. As can be seen in the graph the function has zeros at approximate x-values of 0.1 and 4.1

Steps	Explanation
$\boxed{\text{Range}}$	Enter the x- and y-axis limits.
0 $\boxed{\text{EXE}}$	Xmin becomes 0.
7.5 $\boxed{\text{EXE}}$	Xmax becomes 7.5.
1 $\boxed{\text{EXE}}$	Xscl becomes 1.
$\boxed{(-)}$ 2.5 $\boxed{\text{EXE}}$	Ymin becomes −2.5.
2.5 $\boxed{\text{EXE}}$	Ymax becomes 2.5.
1 $\boxed{\text{EXE}}$	Yscl becomes 1.

We are back at the execution level at this point. If not, use the $\boxed{\text{Range}}$ key again.

$\boxed{\text{Graph}}$	"Y =" appears
2 $\boxed{\sin}$ $\boxed{(}$ $\boxed{\sqrt{\ }}$ $\boxed{(}$ 2 $\boxed{\text{ALPHA}}$ $\boxed{+}$ $\boxed{)}$ $\boxed{)}$ $\boxed{-}$ 0.5 $\boxed{\text{EXE}}$	

It is possible to obtain a better estimate by using the *Trace* feature. For example, select $\boxed{\text{SHIFT}}$ $\boxed{\text{Graph}}$ (trace) and use the $\boxed{\rightarrow}$ and $\boxed{\leftarrow}$ keys to move the blinking dot as close to the point (b) as possible. Then use $\boxed{\text{SHIFT}}$ $\boxed{\times}$ (multiply) to expand the display. It will then look like the lower screen in the preceding figure. Using the trace feature again will show that point (b) is approximately 4.149. (The trace feature shows the current value of x in the display.) By repeating the trace feature we could find a better and better estimate of the zero at (b). By resetting the range and zooming we could repeat the process for the zero at (a). ■

The TI-81 and Newton's Method

Section 5–4 presented solving equations using Newton's method on the TI-81 calculator. The next example solves the previous problem using this method.

Example B

Find the zeros of $y = 2\sin(\sqrt{2x}) - \frac{1}{2}$ for $0 \le x < 2\pi$ (from example A) using Newton's method.

We assume the function is entered into Y_1 as described in example A, and use the trace function to position the cursor near the point (a) in the figure there. Then select $\boxed{\text{PRGM}}$ 1 (assumes the program NEWTON is stored as the first program). The display shows Prgm1 in the display. Use $\boxed{\text{ENTER}}$ to run the program and find the approximate value of x at the point (a) to be 0.0319236557 when the program is finished.

Note *The zoom feature must be used several times before the root at (a) can be found by the program NEWTON.* Not zooming produces an error. This is because the program calculates a value $D = \dfrac{\text{XMAX} - \text{XMIN}}{100}$. This value D is used to compute new values of x. Using XMAX of 7.5 and XMIN of 0 produces a value of D that causes the program to try to use a negative value of x. The value $\sqrt{2x}$ causes a problem when this happens. Rerun the program by resetting the range to its initial values, then selecting $\boxed{\text{GRAPH}}$ and then $\boxed{\text{TRACE}}$ again, placing the cursor near (b). This will show that the zero at (b) is approximately 4.172907423.

Newton's method, used in example B, cannot be conveniently programmed into the Casio calculator. This is because the Texas Instruments calculator has a built-in function called NDeriv that is not available in the Casio. For the Casio, we illustrate another method for approximating zeros of functions—the method of bisection. This method is also suitable for programming on a computer. At the end of this appendix is a Pascal language program for this method.

The Casio *fx*-7000G and the method of bisection

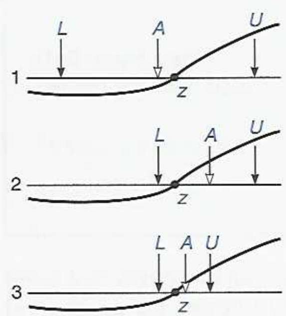


Figure B–1

Figure B–1 illustrates the method of bisection[1] for some hypothetical function. We let z mean zero, L mean lower, U mean upper, and A mean average. Suppose we know there is exactly one zero of the function, at z in the figure, between two values, L and U (part 1 of figure B–1). We can see that $f(L) < 0$ and $f(U) > 0$.

If we let $A = \dfrac{L + U}{2}$ be the average of L and U, A divides the interval between L and U in half. We calculate $f(A)$ and discover that $f(A) < 0$. This means the zero z must be between A and U. This is because the function must go from a negative value at A to a positive value at U, thus taking on the value 0 somewhere in between. Thus, change L to mean the value at c, and repeat the process (part 2 of figure B–1).

[1]To "bisect" means to cut in half.

We now consider the new smaller interval from L to U, and compute the midpoint of this interval, $A = \frac{L + U}{2}$. Since $f(A) > 0$ the zero is between L and A. We therefore let U represent this value of c (part 3 of figure B–1) and repeat the process, obtaining the new point A. We repeat this process until we find a value x so that $f(x) = 0$, or until the interval is so short that its midpoint is a good enough approximation to the zero.

To simplify the program we require $f(L) < 0$ and $f(U) > 0$, as in figure B–1. If this is not true, we interchange the values of L and U. This does not affect the procedure.

An algorithm for this procedure is described below. A program for a Casio *fx*-7000G calculator and a Pascal language program are given at the end of this appendix.

Finding a root of a function by bisection

Assumptions

1. The function has exactly one zero between two values L and U, $L < U$.
2. The function is continuous for all values from L to U inclusive.
3. $f(L) \neq 0$ and $f(U) \neq 0$.
4. $f(L)$ and $f(U)$ have different signs (the root is not of even multiplicity).

Algorithm *{Comments are enclosed in braces.}*

```
Start:  Read in the values L and U.
        If f(L) > 0 then                          {We want f(L) < 0, f(U) > 0}
           interchange L and U.
Loop:   Let A = (L + U)/2.                        {A is the average of L and U}
        If f(A) = 0 then
           go to Finish.                          {A is the zero and we are done}
        If |L - U| < some_predetermined_value then
           go to Finish.
        If f(A) < 0 then                          {See figure B-1}
           Let L = A                              {Part 1 of the figure.}
        otherwise {f(A) > 0 so}
           let U = A.                             {Part 2 of the figure.}
           go to Loop.
Finish: Print out the value of A.
```

The program for the Casio given at the end of this appendix requires two input values of x, which provide an upper (U) and lower (L) bound for an interval that contains a single zero of an equation. These values might easily be found by graphing the equation. The program is stored in, say program 8.

The function for which the zeros are to be calculated is stored as program 9. It must be an expression in the variable X and its last statement must be →Y.

Example C illustrates using this program.

■ ***Example C*** Find the zeros of $y = 2\sin(\sqrt{2x}) - 0.5$ (from example A) using the method of bisection, programmed into program 8 on a Casio *fx*-7000G. (We assume the Casio program that implements the method of bisection has been entered into program 8.)

To enter the equation $y = 2\sin(\sqrt{2x}) - 0.5$ into memory as program 9 proceed as follows.

Key strokes	Comments
MODE 2	Enter WRT (Program WRITE) mode.
Use the → key to select Program 9.	
Then EXE .	
2 sin (√ (2 ALPHA	
+)) − 0.5 →	
ALPHA −	
MODE 1	Go back to run mode.

To run the program to find the zero proceed as follows. Based on figure B–1 we use $L = 0$, $U = 1$.

Key strokes	Comments
Prog 8 EXE	Execute program 8.
?	The ? in the display means to enter a value.
0 EXE	Enter the value 0. This is L.
?	
1 EXE	Enter the value 1. This is U.

0.03192365567 appears when the program is finished.

The zero at (a) in figure B–1 is thus approximately 0.03192365567. Rerunning the program with suitable values for L and U shows that the zero at (b) is approximately 4.172907423. ■

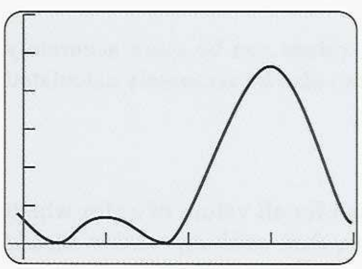


Figure B–2

Note that the program in 8 can be used with any expression that is entered into program 9. The only requirement is that the expression end with "→y."

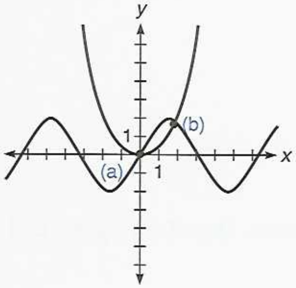
Note If the graph of a function does not cross the x axis at a zero, the zero is said to have even multiplicity. For example, $f(x) = (\sin x - \frac{1}{2})^2$ has a zero at $\frac{\pi}{6}$, but neither the method of bisection nor Newton's method will find it. Figure B–2 shows the graph of this function.

Solving systems of two equations in two variables

A system of two equations in two variables is a collection of two equations, each of which is described in terms of the same two variables, usually x and y. To solve a system of two equations in two variables is to find all ordered pairs (x,y) that satisfy both equations. Graphically, these points are where the graphs intersect. Example D shows two methods to solve these systems with the graphing, programmable calculator.

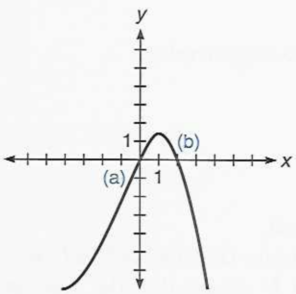
Example D

Solve the system of equations $y = 2 \sin x$ and $y = \frac{1}{2}x^2$.



Method 1: Graph both equations in the same coordinate system. The point(s) of intersection of the graphs are the solutions. In the figure, there are two such points. One is the origin (a) (0,0); the second appears to be approximately (1.9,1.9) (b).

By expanding the graph near this second point we might get a better estimate. A TI-81 displays both the x- and y-values when tracing. On the Casio fx-7000G, use the $\boxed{X\leftrightarrow Y}$ feature to see both the x- and y-values.



Method 2: A second method, which will obtain just the x-value, is to graph the difference of the two x-expressions, in this case $y = (2 \sin x) - (\frac{1}{2}x^2)$.

The points in which we are interested are zeros of this new function. The graph is shown in the figure, where we estimate the x-value of the point at (b) to be 1.9. Using Newton's method or bisection we can quickly find that $x \approx 1.933753763$.

The y-values can be found by evaluating either of the two equations $y = 2 \sin x$ or $y = \frac{1}{2}x^2$ for y using $x = 1.933753763$.

Doing this gives $y \approx 1.869701808$.

The advantage of method 2 is that the x-values can be more accurately estimated, since they fall on the x-axis. They can also be accurately calculated using the methods cited earlier.

Verifying identities

An identity states that two expressions are equal for all values of x (for which each is defined) (chapter 5). If this is the case, then *each expression should have the same graph.*

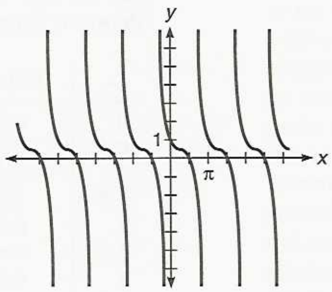
Example E

Show that $\frac{\cos x}{\cos x + \sin x} = \frac{\cot x}{1 + \cot x}$ is probably an identity by graphing each member of the equation.

(This is an identity from section 5–1.)

Graph both $y = \dfrac{\cos x}{\cos x + \sin x}$ and $y = \dfrac{\cot x}{1 + \cot x}$. The figure is the graph of both, which indicates that it is probably an identity.

Note Most calculators and computers do not have the cotangent function built in. Thus, each instance of $\cot x$ must be replaced by $\dfrac{1}{\tan x}$ to graph the expression.



The fact that two graphs look the same is not *proof* that two expressions are identical. A graph is finite and constructed by a machine. For example, on a Casio fx-7000G the graphs of $y = \sin(6x)$ and $y = \sin(100x)$ are the same! This phenomenon is explored in the exercises.

Exercises for Appendix B

a. Graph the following equations for $0 \le x \le 4$.
b. Using the graph, estimate the value of all zeros of the equation for $0 \le x \le 4$.
c. Using the method of bisection or Newton's method find the value of each zero to at least six decimal places.

1. $y = \sin\left(\dfrac{x^2}{2}\right)$ **2.** $y = \sin(\sqrt{3x}) - \frac{1}{2}$ **3.** $y = \sin x - \sin 2x$

4. $y = \sin 3x + \sin 2x$ **5.** $y = 2\sin x - \dfrac{x}{2}$ **6.** $y = \cos 3x + \sin 2x$

Solve the following systems of equations by (a) estimating the solutions from their graphs and then (b) using Newton's method or the method of bisection to find the value of each solution to at least six decimal places.

7. $y = \sin x;\ y = \dfrac{x}{2}$ **8.** $y = \sin 3x;\ y = \sqrt{x}$ **9.** $y = \cos\dfrac{x}{2};\ y = \dfrac{x}{2}$ **10.** $y = x;\ y = \sqrt{x}$

Show that the following are probably identities by graphing the left and right members and observing that the graphs look the same. Use $-2\pi \le x \le 2\pi$ for the domain of the graph.

11. $\csc\theta + \cot\theta = \dfrac{1 + \cos\theta}{\sin\theta}$ **12.** $\tan\theta + \sec\theta = \dfrac{1 + \sin\theta}{\cos\theta}$ **13.** $\dfrac{\csc\theta}{\sec\theta + \tan\theta} = \dfrac{\cos\theta}{\sin\theta + \sin^2\theta}$

14. $\dfrac{\sec\theta}{\csc\theta + \cot\theta} = \dfrac{\tan\theta}{1 + \cos\theta}$ **15.** $\dfrac{1}{\sec\theta - \cos\theta} = \cot\theta\csc\theta$ **16.** $\dfrac{\cot\theta}{\sec\theta} = \csc\theta - \sin\theta$

17. Problems 7 through 19 in appendix A are problems for which a calculator is well suited. Do some of these problems. The results obtained with a calculator should be the same as the answers obtained in the method discussed in appendix A, addition of ordinates.

18. Do problem 20 in appendix A.

19. Do problem 21 in appendix A.

20. Do problem 22 in appendix A.

21. *Fourier series* are functions defined as a sum of an infinite number of terms of the form $a \sin(bx)$ or $a \cos(bx)$. They are used extensively in electronics, music, linguistics, and other areas to analyze wave forms. It turns out that any periodic wave form can be described by an appropriate Fourier series. The four examples in this problem describe "sawtooth" and square-wave forms.

Graph an approximation to each of the following Fourier series by entering at least the first five terms into the calculator. Use the values $-\pi \leq x \leq \pi$ for the graphs. (As more and more terms are used the graphs become more accurate.)

a. $f(x) = \sin x + \frac{1}{2}\sin(2x) + \frac{1}{3}\sin(3x) + \frac{1}{4}\sin(4x) + \cdots$.

b. $f(x) = \sin x + \frac{1}{3}\sin(3x) + \frac{1}{5}\sin(5x) + \frac{1}{7}\sin(7x) + \cdots$.

c. $f(x) = \frac{1}{2}\sin(2x) + \frac{1}{4}\sin(4x) + \frac{1}{6}\sin(6x) + \frac{1}{8}\sin(8x) + \cdots$.

d. $f(x) = \sin x - \frac{1}{2}\sin(2x) + \frac{1}{3}\sin(3x) - \frac{1}{4}\sin(4x) + \cdots$.

$\boxed{\textbf{22.}}$ As stated in this section, on a Casio graphing calculator the graphs of $y = \sin 6x$ and of $y = \sin 100x$ are the same. (At the range settings used for graphing the sine function: Xmin = −6.2831853, Xmax = 6.2831853.) The same phenomenon can occur with any graphing calculator or computer.

To see why this sort of thing can happen, one must realize that the plotting area of most graphing devices is broken down into a grid of points, called pixels,[2] which are either darkened in or left light. The number of these pixels is insufficient for some purposes.

By way of example, assume a situation where pixels represent 0.5 units. (Range settings for which this might occur in the TI-81 and the Casio *fx*-7000G are given below.) This situation is shown in figure B–3. To graph a function we compute values for the function for each value of the domain in increments of 0.5.

[2]Pixel means "picture element."

Table B–1 constructs a set of values for the two functions $y = \sin(x)$ and $y = \sin[(4\pi + 1)x]$, in x-increments of 0.5. These ordered pairs are plotted in figure B–3. Observe that the ordered pairs at the values of x shown have the same y-values, although for other values of x the two functions are not the same. The calculator can only darken complete 0.5 by 0.5 unit squares. When this is done we have the graph in figure B–4. We can see that *either function would produce the graph shown in figure B–4.*

x	$\sin(x)$	$\sin[(4\pi + 1)x]$
0	0.00	0.00
0.5	0.48	0.48
1	0.84	0.84
1.5	1.00	1.00
2	0.91	0.91
2.5	0.60	0.60
3	0.14	0.14
3.5	−0.35	−0.35
4	−0.76	−0.76
4.5	−0.98	−0.98
5	−0.96	−0.96
5.5	−0.71	−0.71
6	−0.28	−0.28
6.5	0.22	0.22

Table B–1

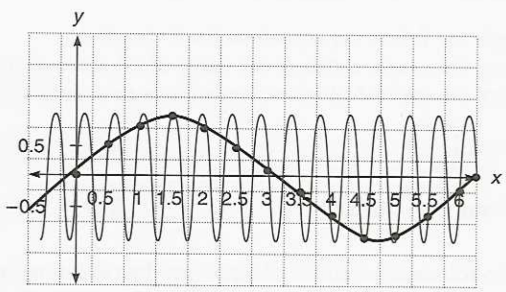


Figure B–3

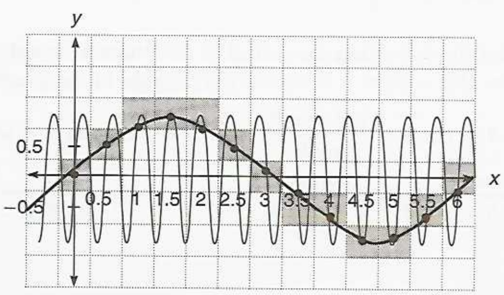


Figure B–4

To investigate this further, consider the following problems.

a. Make a table of values for the following four functions, letting x take on the values 0, 0.5, 1, 1.5, etc., up to 6.0. This is just an extension of table B–1, where the values for the first two functions are already done.

$$y = \sin(x)$$
$$y = \sin[(4\pi + 1)x]$$
$$y = \sin[(8\pi + 1)x]$$
$$y = \sin[(12\pi + 1)x]$$

Try to make a generalization from the results.

Note The four functions will give identical values for a given x, but this does not mean the functions are identical! Just reconsider figure B–3 to see that the first two functions are not identical.

b. Make a table of values for the following four functions, letting x take on the values 0, 0.25, 0.5, 0.75, etc., up to 3. This corresponds to a calculator display with pixels 0.25 units wide.

$$y = \sin(x)$$
$$y = \sin[(8\pi + 1)x]$$
$$y = \sin[(16\pi + 1)x]$$
$$y = \sin[(24\pi + 1)x]$$

Again, look for a pattern.

c. Make a table of values for the following four functions, letting x take on the values 0, 0.1, 0.2, 0.3, etc., up to 1.2. This corresponds to a calculator display 0.1 units wide.

$$\sin(x)$$
$$\sin[(20\pi + 1)x]$$
$$\sin[(40\pi + 1)x]$$
$$\sin[(60\pi + 1)x]$$

Try to find a pattern.

Note On a TI-81 the following settings create pixels of 0.1 units wide:

$$\text{Xmin} = -4.8,\ \text{Xmax} = 4.7,\ \text{Xres} = 1.$$

There are 96 pixels on this calculator, and $\dfrac{4.7 - (-4.8) + 0.1}{96} = 0.1$.

For pixels 0.25 wide, find values so that $\dfrac{\text{Xmas} - \text{Xmin} + 0.25}{96} = 0.25$. A similar calculation can be used to obtain pixels 0.5 units wide.

The Casio *fx*-7000G has 95 pixels, so the settings for creating pixels 0.1 units wide are Xmin = −4.7, Xmax = 4.7. Of course the division shown above then has the value 95 in the denominator.

d. Consider the results of the previous parts of this problem, and try to predict some values β so that $\sin(x)$ and $\sin(\beta x)$ have the same graph on a device that has pixels 0.01 units wide. (That is, that the functions $y = \sin x$ and $y = \sin(\beta x)$ have the same values for $x = 0, 0.01, 0.02, 0.03$, etc.)

e. Show why the generalizations of parts a, b, and c are true (use identities from section 5–2).

Program for the Casio *fx*-7000G calculator to calculate zeros of a function using the method of bisection. It is assumed the function to be solved is defined in program 9.

Generic program	Casio program	Casio key strokes/comments	
		$\boxed{\text{MODE}}$ 2	Enter WRT (Program) mode.
Prepare the calculator to accept a program.		Use the $\boxed{\Rightarrow}$ key to select Program 8.	Actually any program location will do.
		$\boxed{\text{EXE}}$	Ready to enter program 8.
Input the lower value L and the upper value U.	?→L:?→U	$\boxed{\text{SHIFT}}$ $\boxed{\rightarrow}$ $\boxed{\rightarrow}$ $\boxed{\text{ALPHA}}$ $\boxed{\sqrt[x]{\ }}$ $\boxed{:}$ $\boxed{\text{SHIFT}}$ $\boxed{\rightarrow}$ $\boxed{\rightarrow}$ $\boxed{\text{ALPHA}}$ $\boxed{1}$ $\boxed{\text{EXE}}$	

Generic program	Casio program	Casio key strokes/comments
Let $x = L$. Compute $f(L)$. Store the result in A.	L→X:Prog 9:Y→A	ALPHA $\sqrt[x]{\ }$ → ALPHA + : Prog 9 : ALPHA − → ALPHA x^{-1} EXE
If $f(L) < 0$ then go to 1	A<0→Goto 1	ALPHA x^{-1} SHIFT 3 0 SHIFT 7 SHIFT Prog 1 EXE
else switch the values in U and L.	U→T:L→U:T→L	ALPHA 1 → ALPHA ÷ : ALPHA $\sqrt[x]{\ }$ → ALPHA 1 : ALPHA ÷ → ALPHA $\sqrt[x]{\ }$ EXE
1:	Lbl 1	SHIFT ⇐ 1 EXE
Let $X = \dfrac{L+U}{2}$	(L+U)÷2→X	(ALPHA $\sqrt[x]{\ }$ + ALPHA 1) ÷ 2 → ALPHA + EXE
Compute $f(x)$; store result in Y.	Prog 9	Prog 9 EXE Assumes the equation to be solved will be in 9.
If $f(A) = 0$ then go to 3	Y=0→Goto 3	ALPHA − SHIFT 8 0 SHIFT 7 SHIFT Prog 3 EXE
If $\lvert U - L \rvert \le$ error then go to 3	Abs (U−L)≤1 E−11 →Goto 3	SHIFT x^y (ALPHA 1 − ALPHA $\sqrt[x]{\ }$) SHIFT 6 1 EXP (−) 1 1 SHIFT 7 SHIFT Prog 3 EXE
If $Y > 0$ go to 2	Y>0→Goto 2	ALPHA − SHIFT 2 0 SHIFT 7 SHIFT Prog 2 EXE
else let $L = x$ go to 1	X→L:Goto 1	ALPHA + → ALPHA $\sqrt[x]{\ }$: SHIFT Prog 1 EXE
2:	Lbl 2	SHIFT ⇐ 2 EXE
let $U = x$ go to 1	X→U:Goto 1	ALPHA + → ALPHA 1 : SHIFT Prog 1 EXE
3:	Lbl 3	SHIFT ⇐ 3 EXE
Display the final value, x.	X	ALPHA +
Stop the program.		MODE 1 Go back to run mode.

Pascal language program that implements the method of bisection for finding approximations to zeros of functions.

```
program    BISECTION (input,output);
const      ERROR = 1.0E-6;               { Practical smallest difference of
                                           two real data values in Pascal. }
var        Lower, Upper, Average : real;
           Done : boolean;
{ ********** The function for which we want to approximate zeros. ********* }
function   F (x : real) : real;
           begin
               F := 2*sin(sqrt(2 * x)) - 0.5        { DEFINE FUNCTION HERE }
           end { F };
{ ****************************************************************** }
procedure  Read_in (var L, U : real);
           begin
               Write('Enter Lower bound, then Upper bound: ');
               ReadLn(L, U);
           end { Read_in };
procedure  Interchange (var L, U : real);
           var temp : real;
           begin
               temp := L;
               L := U;
               U := temp
           end { Interchange };
begin    { BISECTION }
    Done := FALSE;
    Read_in(Lower, Upper);
    If F(Lower) > 0 then Interchange(Lower, Upper);
    While not Done do begin
        Average := (Lower + Upper) / 2;
        If (f(Average) = 0) or (abs(Upper–Lower) < ERROR) then
            Done := TRUE
        else { Average is not a zero and is not within ERROR units of a zero }
            If f(Average) < 0 then Lower := Average
            else { f(Average) > 0 } Upper := Average
        end { while };
    WriteLn(Average);
end { BISECTION }.
```

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* Undergraduate and graduate borrowers may borrow annually up to the lesser of the cost of attendance or $30,000 ($40,000 for certain schools where it has been determined that the annual cost of attendance exceeds $30,000). Borrowers in the Continuing Education loan program may borrow annually up to $30,000.

** Undergraduate students may choose to defer repayment until six months after graduation or ceasing to be enrolled at least half time in school. Interest only and immediate repayment options also available.

*** A 0.25% interest rate reduction is available for borrowers who elect to have monthly principal and interest payments transferred electronically from a savings or checking account. The interest rate reduction will begin when automatic principal and interest payments start, and will remain in effect as long as automatic payments continue without interruption. This reduced interest rate will return to contract rate if automatic payments are cancelled, rejected or returned for any reason. Upon request, borrowers are also entitled to an additional 0.25% interest rate reduction if (1) the first 36 payments of principal and interest are paid on time, and (2) at any time prior to the 36th on time payment, the borrower who receives the monthly bill elects to have monthly principal and interest payments transferred electronically from a savings or checking account, and continues to make such automatic payments through the 36th payment. This reduced interest rate will not be returned to contract rate if, after receiving the benefit, the borrower discontinues automatic electronic payment. The lender and servicer reserve the right to modify or discontinue borrower benefit programs (other than the co-signer release benefit) at any time without notice.

GMAC Bank Member FDIC

Appendix C

Development of the Identity $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

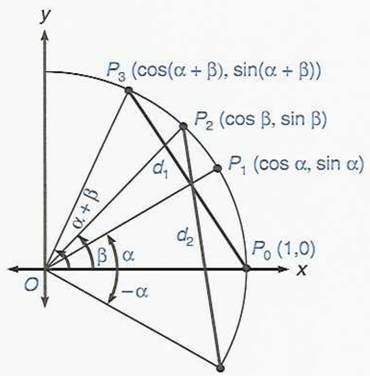


Figure C–1

A proof that this identity is true is beyond the scope of this text, but an argument for its correctness can be obtained in the following way.

Let α and β be two angles in standard position (see figure C.1). Let P_1 be the point where the terminal side of α intersects the unit circle, and P_2 be the point where angle β intersects the unit circle. Let P_3 be the point where the angle $\alpha + \beta$ (the sum of the angles α and β) intersects the circle. Let P_0 be the point (1,0). Finally, let P_4 be the point where the terminal side of angle $-\alpha$ intersects the unit circle.

On the unit circle the x- and y-coordinates of a point are the cosine and sine values for the appropriate angle. Thus, the point P_1 has coordinates $(\cos\alpha, \sin\alpha)$. The coordinates for the other points are shown in the figure.

Angle $\alpha + \beta$, or angle P_0OP_3 in standard position, has the same measure as angle P_4OP_2. It is a geometric property that central angles of a circle having equal measure will have chords of equal length. Thus, the chords P_3P_0 and P_2P_4 have the same length. The length of a line segment with end points (x_1,y_1) and (x_2,y_2) is given by the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

We apply this to the chords mentioned above.
Let d_1 = length of P_3P_0, and d_2 = length of P_2P_4.

$$d_1 = d_2$$

$$\sqrt{(\cos(\alpha + \beta) - 1)^2 + (\sin(\alpha + \beta) - 0)^2} = \sqrt{(\cos\beta - \cos\alpha)^2 + (\sin\beta - (-\sin\alpha))^2}$$

We now square both sides.

$$[\cos(\alpha + \beta) - 1]^2 + [\sin(\alpha + \beta) - 0]^2 = (\cos\beta - \cos\alpha)^2 + [\sin\beta - (-\sin\alpha)]^2$$

Performing the indicated operations we obtain

$$\cos^2(\alpha + \beta) - 2\cos(\alpha + \beta) + 1 + \sin^2(\alpha + \beta) = \cos^2\beta - 2\cos\alpha\cos\beta + \cos^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \sin^2\alpha$$

Then

$$[\cos^2(\alpha + \beta) + \sin^2(\alpha + \beta)] - 2\cos(\alpha + \beta) + 1 = (\cos^2\beta + \sin^2\beta) + (\cos^2\alpha + \sin^2\alpha) + 2\sin\alpha\sin\beta - 2\cos\alpha\cos\beta$$

Using the fundamental identity $\sin^2\theta + \cos^2\theta = 1$, we obtain

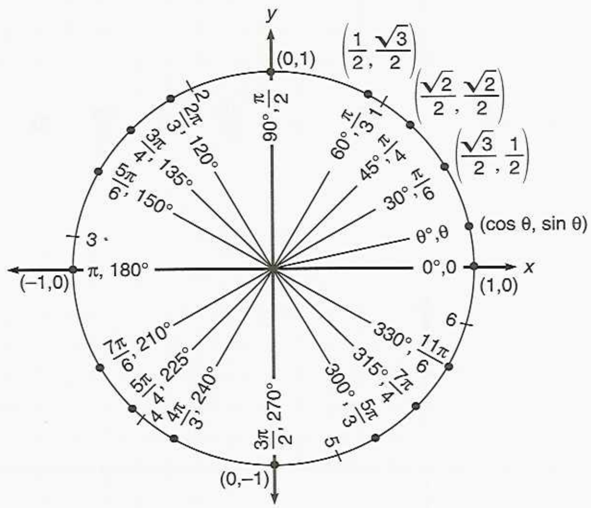
$$1 - 2\cos(\alpha + \beta) + 1 = 1 + 1 + 2\sin\alpha\sin\beta - 2\cos\alpha\cos\beta$$
$$-2\cos(\alpha + \beta) = 2\sin\alpha\sin\beta - 2\cos\alpha\cos\beta$$
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \text{Divide each member by } -2$$

Appendix D

Useful Templates

This appendix includes items that it might prove useful to reproduce in quantity and have handy. Note that rectangular coordinate graph paper and polar coordinate paper are both widely available in book stores.

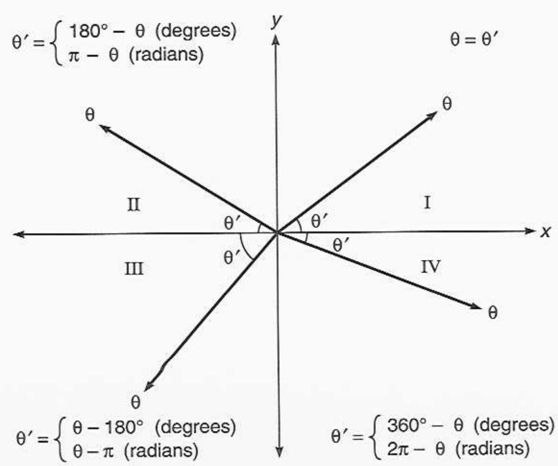
The Unit Circle



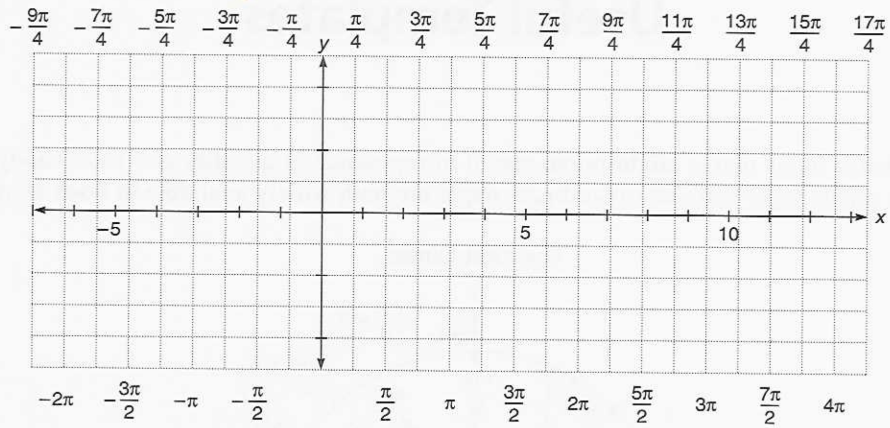
Reference Angles

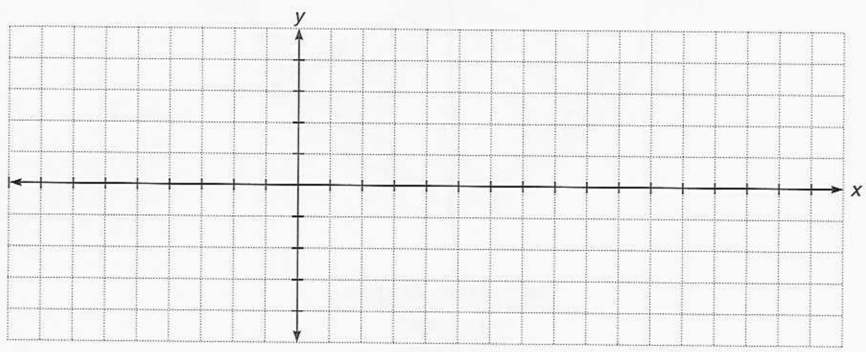
$0 < \theta < 360°$

$0 < \theta < 2\pi$

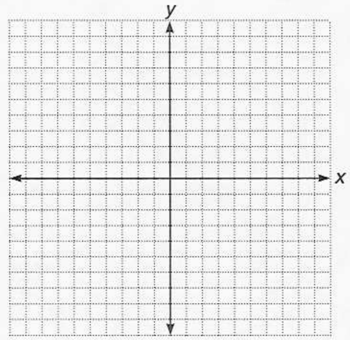


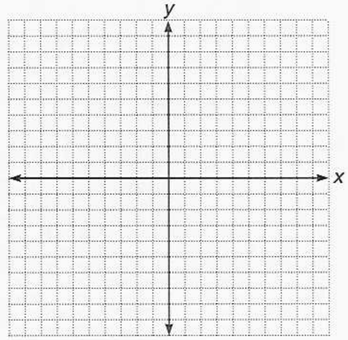
Templates Useful for Graphing Many Trigonometric Functions

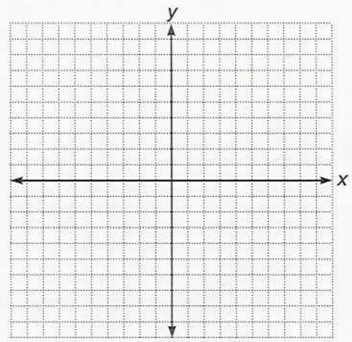


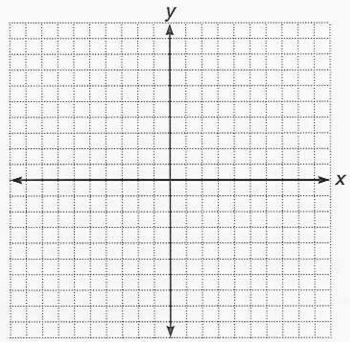


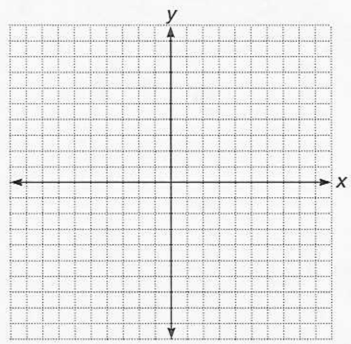
Rectangular Coordinates

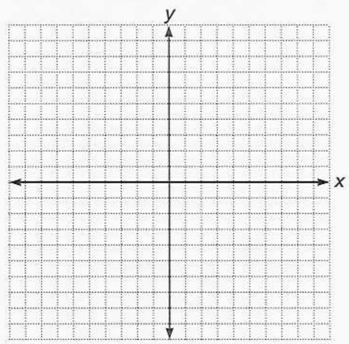




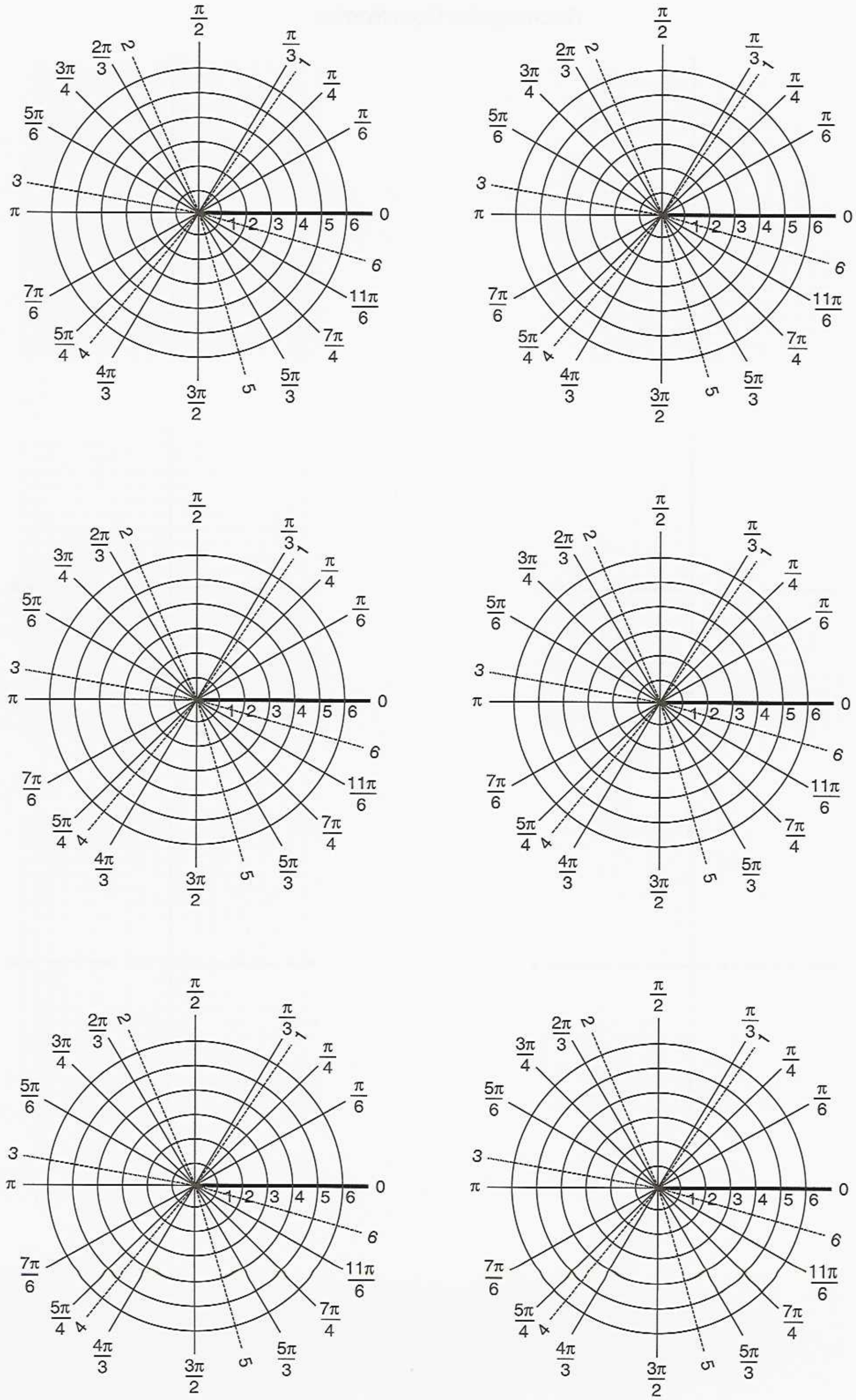








Polar Coordinates



Appendix E

Answers and Solutions

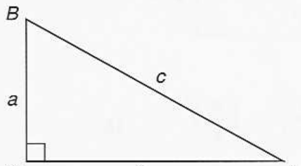
Chapter 1

Exercise 1–1

Answers to odd-numbered problems

1. 13.417°, acute **3.** 0.2°, acute
5. 25.555°, acute **7.** 165.783°, obtuse
9. 33.099°, acute **11.** 159.983°, obtuse

13.



15. 48.2° **17.** 71°48′ **19.** 106°
21. right triangle, hypotenuse = 0.5
23. right triangle, hypotenuse = $2\sqrt{5}$
25. right triangle, hypotenuse = 2.57
27. $c = 15$ **29.** $a = 6$
31. $b = 6\sqrt{5} \approx 13$ **33.** $c = \sqrt{14} \approx 4$
35. $c = 50\sqrt{13} \approx 180$ **37.** $c = 4$
39. $b = \sqrt{185.31} \approx 13.6$
41. $c = 7\sqrt{2} \approx 9.9$ **43.** $a = 3\sqrt{47} \approx 21$
45. $c = \sqrt{2} \approx 1$ **47.** 61 ft **49.** 90.1 ft
51. 20.07 ohms **53.** 3,770 ohms
55. Not accurate, measurements do not satisfy Pythagorean theorem
57. 23 minutes **59.** no, 1.1 ft
61. 107 knots **63.** 17 knots
65. 27 units

Solutions to trial exercise problems

6. $$87°2'13'' = \left(87 + \frac{2}{60} + \frac{13}{3{,}600}\right)^{\circ}$$
$$= (87 + 0.0333 + 0.0036)°$$
$$= 87.037°$$

This angle is acute since it is less than 90°.

18.
$$\begin{array}{rr} 28° & 17' \\ +\ 28° & 52' \\ \hline 56° & 69' \\ 56°\ 60' & +\ 9' \\ 57° & 9' \\ \\ 180° & \\ -\ 57° & 9' \\ \hline 179° & 60' \\ -\ 57° & 9' \\ \hline 122° & 51' \end{array}$$

The angle is 122°51′.

20.
$$\begin{array}{r} 90° \\ -72°\ 19'\ 51'' \\ \hline 89°\ 60' \\ -72°\ 19'\ 51'' \\ \hline 89°\ 59'\ 60'' \\ -72°\ 19'\ 51'' \\ \hline 17°\ 40'\ 9'' \end{array}$$

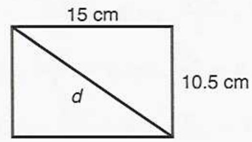
23. $(2\sqrt{5})^2 = 2^2(\sqrt{5})^2 = 4(5) = 20$;
$2^2 + 4^2 = 4 + 16 = 20$.
Since $(2\sqrt{5})^2 = 2^2 + 4^2$, the triangle is a right triangle; the hypotenuse is the longest side, with length $2\sqrt{5}$.

29. $a^2 + b^2 = c^2$, so $a^2 + 8^2 = 10^2$
$$a^2 + 64 = 100$$
$$a^2 = 36$$
$$a = 6$$

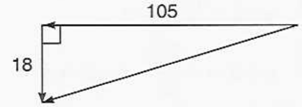
41. $c^2 = a^2 + b^2$
$$c^2 = (3\sqrt{2})^2 + (4\sqrt{5})^2$$
$$= 9(2) + 16(5) = 98$$
$$c = \sqrt{98} = \sqrt{2 \cdot 49} = 7\sqrt{2}$$

54. The sum of the angles is 110°0′ + 33°28′ + 28°32′ = 172°. Since the sum of the angles of a triangle is 180°, we can see that there is quite a bit of inaccuracy in these measurements. Although in practice we would not expect the three measurements to add exactly to 180° (they are approximations, as are any measurements), an error of 8° out of 180° (more than 4% error) probably indicates an error in a measurement.

57. The diagonal is $\sqrt{15^2 + 10.5^2} \approx 18.31$ inches. (See the diagram.) Divide 18.31 inches by 0.8 inch per minute to get 23 minutes.



61. The headings of the aircraft and the wind form the wind triangle shown.

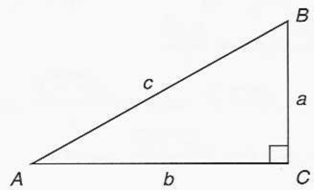


Ground speed = $\sqrt{105^2 + 18^2} \approx 107$ knots

Exercise 1–2

Answers to odd-numbered problems

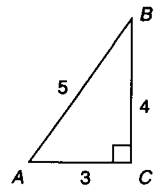
1.



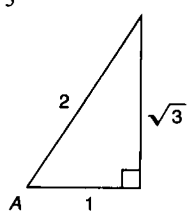
$$\sin A = \frac{a}{c},\ \cos A = \frac{b}{c},\ \tan A = \frac{a}{b},$$
$$\cot A = \frac{b}{a},\ \sec A = \frac{c}{b},\ \csc A = \frac{c}{a},$$
$$\sin B = \frac{b}{c},\ \cos B = \frac{a}{c},\ \tan B = \frac{b}{a},$$
$$\cot B = \frac{a}{b},\ \sec B = \frac{c}{a},\ \csc B = \frac{c}{b}$$

3. $c = 5$, $\sin B = \frac{4}{5}$, $\cos B = \frac{3}{5}$, $\tan B = \frac{4}{3}$, $\csc B = \frac{5}{4}$, $\sec B = \frac{5}{3}$, $\cot B = \frac{3}{4}$
5. $c = \sqrt{10}$, $\sin B = \dfrac{3\sqrt{10}}{10}$, $\cos B = \dfrac{\sqrt{10}}{10}$, $\tan B = 3$, $\csc B = \dfrac{\sqrt{10}}{3}$, $\sec B = \sqrt{10}$, $\cot B = \frac{1}{3}$
7. $c = 4\sqrt{2}$, $\sin B = \dfrac{\sqrt{14}}{8}$, $\cos B = \dfrac{5\sqrt{2}}{8}$, $\tan B = \dfrac{\sqrt{7}}{5}$, $\csc B = \dfrac{4\sqrt{14}}{7}$, $\sec B = \dfrac{4\sqrt{2}}{5}$, $\cot B = \dfrac{5\sqrt{7}}{7}$
9. $b = 1$, $\sin B = \dfrac{\sqrt{5}}{5}$, $\cos B = \dfrac{2\sqrt{5}}{5}$, $\tan B = \frac{1}{2}$, $\csc B = \sqrt{5}$, $\sec B = \dfrac{\sqrt{5}}{2}$, $\cot B = 2$
11. $c = \sqrt{313}$, $\sin B = \dfrac{13\sqrt{313}}{313}$, $\cos B = \dfrac{12\sqrt{313}}{313}$, $\tan B = \frac{13}{12}$, $\csc B = \dfrac{\sqrt{313}}{13}$, $\sec B = \dfrac{\sqrt{313}}{12}$, $\cot B = \frac{12}{13}$
13. $b = 8$, $\sin A = \frac{3}{5}$, $\cos A = \frac{4}{5}$, $\tan A = \frac{3}{4}$, $\csc A = \frac{5}{3}$, $\sec A = \frac{5}{4}$, $\cot A = \frac{4}{3}$
15. $c = \sqrt{19}$, $\sin A = \dfrac{\sqrt{57}}{19}$, $\cos A = \dfrac{4\sqrt{19}}{19}$, $\tan A = \dfrac{\sqrt{3}}{4}$, $\csc A = \dfrac{\sqrt{57}}{3}$, $\sec A = \dfrac{\sqrt{19}}{4}$, $\cot A = \dfrac{4\sqrt{3}}{3}$
17. $b = \sqrt{z^2 - x^2}$, $\sin B = \dfrac{\sqrt{z^2 - x^2}}{z}$, $\cos B = \dfrac{x}{z}$, $\tan B = \dfrac{\sqrt{z^2 - x^2}}{x}$, $\csc B = \dfrac{z}{\sqrt{z^2 - x^2}}$, $\sec B = \dfrac{z}{x}$, $\cot B = \dfrac{x}{\sqrt{z^2 - x^2}}$
19. $b = \sqrt{3}$, $\sin B = \dfrac{\sqrt{3}}{2}$, $\cos B = \frac{1}{2}$, $\tan B = \sqrt{3}$, $\csc B = \dfrac{2\sqrt{3}}{3}$, $\sec B = 2$, $\cot B = \dfrac{\sqrt{3}}{3}$

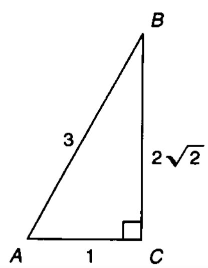
21. $c = \sqrt{106}$, $\sin B = \dfrac{5\sqrt{106}}{106}$, $\cos B = \dfrac{9\sqrt{106}}{106}$, $\tan B = \frac{5}{9}$, $\csc B = \dfrac{\sqrt{106}}{5}$, $\sec B = \dfrac{\sqrt{106}}{9}$, $\cot B = \frac{9}{5}$
23. $\cos A = \frac{3}{5}$, $\tan A = \frac{4}{3}$, $\cot A = \frac{3}{4}$, $\sec A = \frac{5}{3}$, $\csc A = \frac{5}{4}$



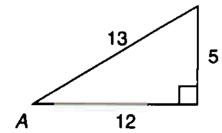
25. $\sin A = \dfrac{\sqrt{3}}{2}$, $\tan A = \sqrt{3}$, $\cot A = \dfrac{\sqrt{3}}{3}$, $\sec A = 2$, $\csc A = \dfrac{2\sqrt{3}}{3}$



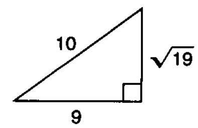
27. $\sin A = \dfrac{2\sqrt{2}}{3}$, $\cos A = \frac{1}{3}$, $\tan A = 2\sqrt{2}$, $\cot A = \dfrac{\sqrt{2}}{4}$, $\csc A = \dfrac{3\sqrt{2}}{4}$



29. $\cos A = \frac{12}{13}$, $\tan A = \frac{5}{12}$, $\cot A = \frac{12}{5}$, $\sec A = \frac{13}{12}$, $\csc A = \frac{13}{5}$

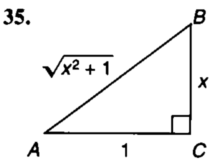


31. $\sin A = \dfrac{\sqrt{19}}{10}$, $\tan A = \dfrac{\sqrt{19}}{9}$, $\cot A = \dfrac{9\sqrt{19}}{19}$, $\sec A = \frac{10}{9}$, $\csc A = \dfrac{10\sqrt{19}}{19}$



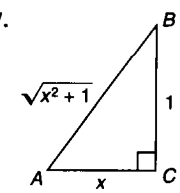
33. $\left(\dfrac{b}{c}\right)^2 + \left(\dfrac{a}{c}\right)^2$
$\dfrac{b^2}{c^2} + \dfrac{a^2}{c^2}$
$\dfrac{b^2 + a^2}{c^2}$
$\dfrac{c^2}{c^2} = 1$

35.



$\dfrac{1}{\sqrt{x^2 + 1}}$

37.



$\dfrac{1}{x}$

Solutions to trial exercise problems

15. $a = \sqrt{3}$, $b = 4$. Find ratios for angle A.

$c^2 = a^2 + b^2$
$c^2 = (\sqrt{3})^2 + 4^2$
$c^2 = 3 + 16 = 19$
$c = \sqrt{19}$

$$\sin A = \frac{a}{c} = \frac{\sqrt{3}}{\sqrt{19}} = \frac{\sqrt{3}}{\sqrt{19}} \cdot \frac{\sqrt{19}}{\sqrt{19}} = \frac{\sqrt{57}}{19};$$

$$\cos A = \frac{b}{c} = \frac{4}{\sqrt{19}} = \frac{4}{\sqrt{19}} \cdot \frac{\sqrt{19}}{\sqrt{19}} = \frac{4\sqrt{19}}{19};$$

$$\tan A = \frac{a}{b} = \frac{\sqrt{3}}{4}; \csc A = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{3}}{\sqrt{19}}} = \frac{\sqrt{19}}{\sqrt{3}}$$

$$= \frac{\sqrt{19}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{57}}{3}; \sec A = \frac{1}{\cos A} = \frac{1}{\frac{4}{\sqrt{19}}} = \frac{\sqrt{19}}{4};$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{\frac{\sqrt{3}}{4}} = \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

19. $a = 1$, $c = 2$. Find ratios for angle B.

$a^2 + b^2 = c^2$, so

$1 + b^2 = 4$

$b^2 = 3$

$b = \sqrt{3}$

$$\sin B = \frac{\text{opp}}{\text{hyp}} = \frac{b}{c} = \frac{\sqrt{3}}{2};$$

$$\cos B = \frac{\text{adj}}{\text{hyp}} = \frac{a}{c} = \frac{1}{2};$$

$$\tan B = \frac{\text{opp}}{\text{adj}} = \frac{b}{a} = \frac{\sqrt{3}}{1} = \sqrt{3};$$

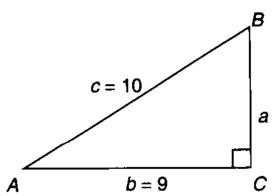
$$\cot B = \frac{1}{\tan B} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3};$$

$$\sec B = \frac{1}{\cos B} = \frac{1}{\frac{1}{2}} = 2;$$

$$\csc B = \frac{1}{\sin B} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3}$$

31. $\cos A = 0.9$

$\cos A = \dfrac{9}{10} = \dfrac{b}{c}$, so a triangle in which $b = 9$ and $c = 10$ would work. This is shown in the diagram.



Using the Pythagorean theorem, we find a.

$c^2 = a^2 + b^2$

$100 = a^2 + 81$

$\sqrt{19} = a$

With this we can now compute the five remaining ratios.

$$\sin A = \frac{a}{c} = \frac{\sqrt{19}}{10};$$

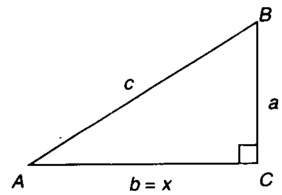
$$\tan A = \frac{a}{b} = \frac{\sqrt{19}}{9};$$

$$\cot A = \frac{1}{\tan A} = \frac{9\sqrt{19}}{19};$$

$$\csc A = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{19}}{10}} = \frac{10\sqrt{19}}{19};$$

$$\sec A = \frac{1}{\cos A} = \frac{1}{\frac{9}{10}} = \frac{10}{9}$$

37. $\tan B = x$; if we think of x as $\dfrac{x}{1}$ we can use the triangle shown in the diagram, since $\tan B = \dfrac{b}{a} = \dfrac{x}{1}$.



From the Pythagorean theorem we find c.

$c^2 = a^2 + b^2$

$c^2 = 1^2 + x^2$ so

$c = \sqrt{x^2 + 1}$

Now we find $\tan A$:

$\tan A = \dfrac{a}{b} = \dfrac{1}{x}$. (Note that we did not actually need c.)

Exercise 1–3

Answers to odd-numbered problems

1. 0.5192 **3.** 0.2116 **5.** 1.8137 **7.** 0.6465 **9.** 2.5048 **11.** 0.9793 **13.** 0.9801 **15.** 0.0790 **17.** 0.7037 **19.** 0.5534 **21.** 0.7447 **23.** 15.7801 **25.** 971.40 **27.** 0.47 **29.** 742.5 watts **31.** 11.8 inches

33. $c^2 = 1^2 + 1^2$

$= 1 + 1$

$= 2$

$c = \sqrt{2}$

$$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45° = \frac{1}{1} = 1$$

35. 53.2° **37.** 62.0° **39.** 31.6° **41.** 31.7° **43.** 63.4° **45.** 78.00° **47.** 68.63° **49.** $A = 51.7°$, $b \approx 12.0$, $c \approx 19.4$ **51.** $B = 76.3°$, $b \approx 45.5$, $c \approx 46.9$ **53.** $B = 60.6°$, $a \approx 0.379$, $c \approx 0.771$ **55.** $A = 12°$, $a \approx 4.6$, $c \approx 22.3$ **57.** $B = 75°$, $a \approx 2.6$, $b \approx 9.7$ **59.** $A = 24.5°$, $a \approx 51$, $b \approx 111$ **61.** $c \approx 20.4$, $A \approx 40.0°$, $B \approx 50.0°$ **63.** $c \approx 1.36$, $A \approx 9.3°$, $B \approx 80.7°$ **65.** $b \approx 17.8$, $A \approx 44.9°$, $B \approx 45.1°$ **67.** $a \approx 98.4$, $A \approx 62.5°$, $B \approx 27.5°$ **69.** $b \approx 5$, $A \approx 67.4°$, $B \approx 22.6°$ **71.** $a \approx 32.6$, $A \approx 21.2°$, $B \approx 68.8°$ **73.** 55.1 ohms **75.** 34° **77.** 18.6 volts **79.** 32,800 ft **81.** 217 mm **83.** 2.3° **85.** 265.8 **87.** $A = 30°$, $a = \dfrac{8\sqrt{3}}{3}$, $c = \dfrac{16\sqrt{3}}{3}$

Solutions to trial exercise problems

9. $\sec 66.47° = \dfrac{1}{\cos 66.47°}$

66.47 [cos] [1/x]

[TI-81] [(] [COS] 66.47 [)] [x^{-1}] [ENTER]

≈ 2.5048 Display: [2.50482689]

Make sure calculator is in degree mode.

11. sin 78.33°

78.33 [sin]

[TI-81] [SIN] 78.33 [ENTER]

≈ 0.9793 Display: [0.9793288556]

Make sure calculator is in degree mode.

13. sin 78°33′

Ⓐ 78 [+] 33 [÷] 60 [=] [sin]

Ⓟ 78 [ENTER] 33 [ENTER] 60 [÷] [+] [sin]

[TI-81] [SIN] [(] 78 [+] 33 [÷] 60 [)] [ENTER]

≈ 0.9801 Display: [0.9800983128]

Make sure calculator is in degree mode.

25. $R = \dfrac{LC}{2 \sin I}$, $LC = 611.1$ meters, $I = 18°20'$.

$R = \dfrac{611.1}{2 \sin 18°20'} \approx \dfrac{611.1}{2(0.3145)} \approx \dfrac{611.1}{0.6290} \approx 971.40$

27. $y = x \cos A \cos B - x^2 \cos A \sin B - x^3 \sin A$

$= 1.2 \cos 10° \cos 15° - 1.2^2 \cos 10° \sin 15° - 1.2^3 \sin 10°$

≈ 0.47 (to two decimal places)

Ⓐ 1.2 $\boxed{\times}$ 10 $\boxed{\text{COS}}$ $\boxed{\times}$ 15 $\boxed{\text{COS}}$ $\boxed{-}$

1.2 $\boxed{x^2}$ $\boxed{\times}$ 10 $\boxed{\text{COS}}$ $\boxed{\times}$ 15 $\boxed{\text{SIN}}$ $\boxed{-}$

1.2 $\boxed{y^x}$ 3 $\boxed{\times}$ 10 $\boxed{\text{SIN}}$ $\boxed{=}$

Ⓟ 1.2 $\boxed{\text{ENTER}}$ 10 $\boxed{\text{COS}}$ $\boxed{\times}$ 15 $\boxed{\text{COS}}$ $\boxed{\times}$

1.2 $\boxed{x^2}$ 10 $\boxed{\text{COS}}$ $\boxed{\times}$ 15 $\boxed{\text{SIN}}$ $\boxed{\times}$ $\boxed{-}$

1.2 $\boxed{\text{ENTER}}$ 3 $\boxed{y^x}$ 10 $\boxed{\text{SIN}}$ $\boxed{\times}$ $\boxed{-}$

$\boxed{\text{TI-81}}$ 1.2 $\boxed{\text{STO}\blacktriangleright}$ $\boxed{\text{X|T}}$ $\boxed{\text{ENTER}}$ 10 $\boxed{\text{STO}\blacktriangleright}$ $\boxed{\text{MATH}}$ $\boxed{\text{ENTER}}$ 15 $\boxed{\text{STO}\blacktriangleright}$ $\boxed{\text{MATRX}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{X|T}}$ $\boxed{\text{COS}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{MATH}}$ $\boxed{\text{COS}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{MATRX}}$ $\boxed{-}$ $\boxed{\text{X|T}}$ $\boxed{x^2}$ $\boxed{\text{COS}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{MATH}}$ $\boxed{\text{SIN}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{MATRX}}$ $\boxed{-}$ $\boxed{\text{X|T}}$ $\boxed{\text{MATH}}$ 3 $\boxed{\text{SIN}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{MATH}}$ $\boxed{\text{ENTER}}$

31. $r = \dfrac{f}{2 \cos \theta} = \dfrac{21.4}{2 \cos 25°} \approx \dfrac{21.4}{1.8126} \approx 11.8$ inches

47. $\sec \theta = \dfrac{6.45}{2.35}$

$\cos \theta = \dfrac{2.35}{6.45}$

$\theta = \cos^{-1} \dfrac{2.35}{6.45}$

$\theta \approx 68.63°$ Display: $\boxed{68.6329624}$

2.35 $\boxed{\div}$ 6.45 $\boxed{=}$ $\boxed{\cos^{-1}}$

$\boxed{\text{TI-81}}$ $\boxed{\text{COS}^{-1}}$ $\boxed{(}$ 2.35 $\boxed{\div}$ 6.45 $\boxed{)}$ $\boxed{\text{ENTER}}$

Make sure calculator is in degree mode.

Remember, $\boxed{\cos^{-1}}$ is $\boxed{\text{SHIFT}}$ $\boxed{\cos}$ or $\boxed{\text{2nd}}$ $\boxed{\cos}$ on most calculators.

48. $\sin \theta = \dfrac{1}{\sqrt{10.8}}$

$\theta = \sin^{-1} \dfrac{1}{\sqrt{10.8}}$

$\theta \approx 17.72°$ Display: $\boxed{17.71547234}$

10.8 $\boxed{\sqrt{x}}$ $\boxed{1/x}$ $\boxed{\sin^{-1}}$

$\boxed{\text{TI-81}}$ $\boxed{\text{SIN}^{-1}}$ $\boxed{(}$ $\boxed{\sqrt{\ }}$ 10.8 $\boxed{)}$ $\boxed{x^{-1}}$ $\boxed{\text{ENTER}}$

Make sure calculator is in degree mode.

Remember, $\boxed{\sin^{-1}}$ is $\boxed{\text{SHIFT}}$ $\boxed{\sin}$ or $\boxed{\text{2nd}}$ $\boxed{\sin}$ on most calculators.

49. $a = 15.2$, $B = 38.3°$

$A = 90° - 38.3° = 51.7°$

Using angle B: $\sin B = \dfrac{b}{c}$, $\cos B = \dfrac{a}{c}$, $\tan B = \dfrac{b}{a}$

$\sin 38.3° = \dfrac{b}{c}$, $\cos 38.3° = \dfrac{15.2}{c}$, $\tan 38.3° = \dfrac{b}{15.2}$

Use the cosine and tangent ratios:

$\cos 38.3° = \dfrac{15.2}{c}$, $\tan 38.3° = \dfrac{b}{15.2}$; $c = \dfrac{15.2}{\cos 38.3°}$,

$15.2(\tan 38.3°) = b$; $c \approx 19.4$, $12.0 \approx b$

59. $c = 122$, $B = 65.5°$

$A = 90° - 65.5° = 24.5°$

$\sin 65.5° = \dfrac{b}{122}$; $\cos 65.5° = \dfrac{a}{122}$; $\tan 65.5° = \dfrac{b}{a}$

$b = 122 \sin 65.5° \approx 111$; $a = 122 \cos 65.5° \approx 51$

67. $b = 51.3$, $c = 111.0$

$c^2 = a^2 + b^2$; $111^2 = a^2 + 51.3^2$; $a \approx 98.4$

$\sin A = \dfrac{a}{111.0}$; $\cos A = \dfrac{51.3}{111.0}$; $\tan A = \dfrac{51.3}{a}$

Using $\cos A = \dfrac{51.3}{111.0}$, we find $A \approx 62.5°$

$B = 90° - 62.5° \approx 27.5°$

76. Construct a line perpendicular to one side b, as shown. Since $b = 8.25''$, half that is $4.125''$.

Since $\theta = \dfrac{360°}{7}$, $\dfrac{\theta}{2} = \dfrac{1}{2} \cdot \dfrac{360°}{7} = \dfrac{180°}{7}$.

$\sin \dfrac{\theta}{2} = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{4.125}{a}$

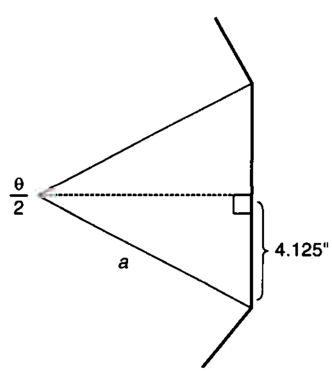
$\sin \dfrac{180°}{7} = \dfrac{4.125}{a}$

$a = \dfrac{4.125}{\sin \dfrac{180°}{7}} \approx 9.51$

4.125 $\boxed{\div}$ $\boxed{(}$ 180 $\boxed{\div}$ 7 $\boxed{)}$ $\boxed{\sin}$ $\boxed{=}$

Display: $\boxed{9.507155093}$

$\boxed{\text{TI-81}}$ 4.125 $\boxed{\div}$ $\boxed{\text{SIN}}$ $\boxed{(}$ 180 $\boxed{\div}$ 7 $\boxed{)}$ $\boxed{\text{ENTER}}$



87. $B = 60°, b = 8$
$A = 90° - 60° = 30°$
$\sin 60° = \frac{8}{c}$, so $c = \frac{8}{\sin 60°} = \frac{8}{\frac{\sqrt{3}}{2}} = \frac{8}{1} \cdot \frac{2}{\sqrt{3}} = \frac{16\sqrt{3}}{3}$
$\tan 60° = \frac{8}{a}$, so $a = \frac{8}{\tan 60°} = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$

Exercise 1–4

Answers to odd-numbered problems

1. $\tan\theta \cot\theta$
$\frac{1}{\cot\theta}\cot\theta$
1

3. $\cos\theta(1 - \sec\theta)$
$\cos\theta\left(1 - \frac{1}{\cos\theta}\right)$
$\cos\theta - \frac{\cos\theta}{\cos\theta}$
$\cos\theta - 1$

5. $\sec\theta(\cot\theta + \cos\theta - 1)$
$\sec\theta\cot\theta + \sec\theta\cos\theta - \sec\theta$
$\frac{1}{\cos\theta}\frac{\cos\theta}{\sin\theta} + \frac{1}{\cos\theta}\cos\theta - \sec\theta$
$\frac{1}{\sin\theta} + 1 - \sec\theta$
$\csc\theta + 1 - \sec\theta$

7. $\frac{\cos\alpha - \sin\alpha}{\cos\alpha}$
$\frac{\cos\alpha}{\cos\alpha} - \frac{\sin\alpha}{\cos\alpha}$
$1 - \tan\alpha$

9. $1 - \cos^2\theta$
$\sin^2\theta + \cos^2\theta - \cos^2\theta$
$\sin^2\theta$

11. $\cos\beta(\sec\beta - \cos\beta)$
$\cos\beta\sec\beta - \cos^2\beta$
$\cos\beta\frac{1}{\cos\beta} - \cos^2\beta$
$1 - \cos^2\beta$
$\sin^2\beta + \cos^2\beta - \cos^2\beta$
$\sin^2\beta$

13. $(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) + 2\sin^2\theta$
$\cos^2\theta + \cos\theta\sin\theta - \cos\theta\sin\theta - \sin^2\theta + 2\sin^2\theta$
$\cos^2\theta + \sin^2\theta$
1

15. a. $\sin^2 16°50' + \cos^2 16°50'$
$\sin^2 16.83° + \cos^2 16.83°$
$0.0838 + 0.9162$
1

b. $\sin^2 50° + \cos^2 50°$
$0.5868 + 0.4132$
1

17. $\tan 32°40' \approx 0.6412$
$\frac{\sin 32°40'}{\cos 32°40'} \approx \frac{0.5398}{0.8418} \approx 0.6412$

19. $\tan x(\cot x + \csc x) = 1 + \sec x$
$\tan x \cdot \cot x + \tan x \cdot \csc x$
$\tan x \cdot \frac{1}{\tan x} + \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}$
$1 + \frac{1}{\cos x}$
$1 + \sec x$

21. $\sin\beta(\cot\beta - \csc\beta + \sin\beta) = \cos\beta - \cos^2\beta$
$\sin\beta \cdot \cot\beta - \sin\beta \cdot \csc\beta + \sin\beta \cdot \sin\beta$
$\sin\beta \cdot \frac{\cos\beta}{\sin\beta} - \sin\beta \cdot \frac{1}{\sin\beta} + \sin^2\beta$
$\cos\beta - 1 + \sin^2\beta$
$\cos\beta - (\cos^2\beta + \sin^2\beta) + \sin^2\beta$
$\cos\beta - \cos^2\beta$

23. $\cos\alpha(\csc\alpha + \sec\alpha) = \cot\alpha + 1$
$\cos\alpha \cdot \csc\alpha + \cos\alpha \cdot \sec\alpha$
$\cos\alpha \cdot \frac{1}{\sin\alpha} + \cos\alpha \cdot \frac{1}{\cos\alpha}$
$\frac{\cos\alpha}{\sin\alpha} + 1$
$\cot\alpha + 1$

25. 60° **27.** 60° **29.** 11.5° **31.** 77.5° **33.** 33.1° **35.** 10° **37.** 30° **39.** 6.5° **41.** 15° **43.** 25.3°

45. $\sin B = \frac{b}{c}$, $\cos B = \frac{a}{c}$, $\tan B = \frac{b}{a}$
$\frac{b}{a} = \tan B = \frac{\sin B}{\cos B} = \frac{\frac{b}{c}}{\frac{a}{c}} = \frac{b}{c} \cdot \frac{c}{a} = \frac{b}{a}$

Solutions to trial exercise problems

8. $\frac{\sin\theta + \cos\theta - 2}{\cos\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{2}{\cos\theta}$
$= \tan\theta + 1 - 2\sec\theta.$

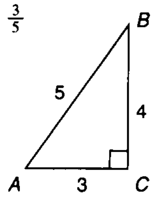
26. $\sqrt{3}\tan x = 1$
$\tan x = \frac{1}{\sqrt{3}}$
$\tan x = \frac{\sqrt{3}}{3}$
$x = \tan^{-1}\frac{\sqrt{3}}{3}$
$x = 30°$, since $\tan 30° = \frac{\sqrt{3}}{3}$

42. $3\sin 2x = 0.75$
$\sin 2x = 0.25$
$2x = \sin^{-1} 0.25$
$x = \frac{\sin^{-1} 0.25}{2}$
$x \approx 7.2°$
0.25 $\boxed{\sin^{-1}}$ $\boxed{\div}$ 2 $\boxed{=}$
Display: $\boxed{7.238756093}$
$\boxed{\text{TI-81}}$ $\boxed{\text{SIN}^{-1}}$ .25 $\boxed{\div}$ 2 $\boxed{\text{ENTER}}$

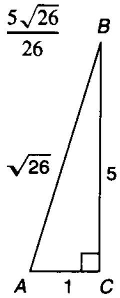
Chapter 1 review

1. 17.57°, acute **2.** 84.15°, acute **3.** 125.62°, obtuse **4.** 39.76°, acute **5.** 59°56′52″ **6.** 61.3° **7.** $4\sqrt{13} \approx 14.4$ **8.** $\sqrt{595} \approx 24.4$ **9.** $\sqrt{105} \approx 10.2$ **10.** 52 ft **11.** 40.0 ohms **12.** $c = \sqrt{58}$, $\sin A = \frac{3\sqrt{58}}{58} \approx 0.3939$, $\cos A = \frac{7\sqrt{58}}{58} \approx 0.9191$, $\tan A = \frac{3}{7} \approx 0.4286$, $\csc A = \frac{\sqrt{58}}{3} \approx 2.5386$, $\sec A = \frac{\sqrt{58}}{7} \approx 1.0880$, $\cot A = \frac{7}{3} \approx 2.3333$ **13.** $b = 10\sqrt{2}$, $\cos B = \frac{1}{3} \approx 0.333$, $\sin B = \frac{2\sqrt{2}}{3} \approx 0.9428$, $\cot B = \frac{\sqrt{2}}{4} \approx 0.3536$, $\sec B = 3$, $\csc B = \frac{3\sqrt{2}}{4} \approx 1.0607$, $\tan B = 2\sqrt{2} \approx 2.8284$ **14.** $c = \sqrt{14}$, $\sin A = \frac{\sqrt{14}}{7} \approx 0.5345$, $\cos A = \frac{\sqrt{35}}{7} \approx 0.8452$, $\tan A = \frac{\sqrt{10}}{5} \approx 0.6325$, $\csc A = \frac{\sqrt{14}}{2} \approx 1.8708$, $\sec A = \frac{\sqrt{35}}{5} \approx 1.1832$, $\cot A = \frac{\sqrt{10}}{2} \approx 1.5811$

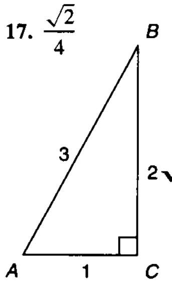
15. $\frac{3}{5}$



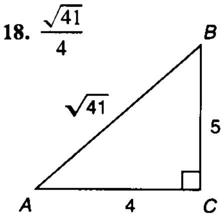
16. $\frac{5\sqrt{26}}{26}$



17. $\frac{\sqrt{2}}{4}$



18. $\frac{\sqrt{41}}{4}$



19. 0.2672 **20.** 2.6927 **21.** 0.3230
22. 0.6534 **23.** 0.2924 **24.** 1.1098
25. 1.6426 **26.** 13.2347 **27.** 0.6128
28. 1.4617 **29.** 2.0057 **30.** 17.1984
31. a. $\sin 53.20° \approx 0.8007$, $\cos 53.20° \approx 0.5990$, $\tan 53.20° \approx 1.3367$, $\csc 53.20° \approx 1.2489$, $\sec 53.20° \approx 1.6694$, $\cot 53.20° \approx 0.7481$ **b.** $\sin 53°20' \approx 0.8021$, $\cos 53°20' \approx 0.5972$, $\tan 53°20' \approx 1.3432$, $\csc 53°20' \approx 1.2467$, $\sec 53°20' \approx 1.6746$, $\cot 53°20' \approx 0.7445$ **32.** 410.101
33. 39.2° **34.** 82.1° **35.** 35.7°
36. 64.8° **37.** 44.6° **38.** 9.0°
39. $c \approx 28.1$, $b \approx 15.7$, $B \approx 33.9°$
40. $a \approx 20.0$, $A \approx 60.3°$, $B \approx 29.7°$
41. $c \approx 9.44$, $A \approx 25.1°$, $B \approx 64.9°$
42. $b \approx 58.9$, $a \approx 29.9$, $A \approx 26.9°$
43. 4.39 km **44.** 36.0 mm
45. $\sin^2 45 + \cos^2 45$

$$\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2$$
$$\tfrac{2}{4} + \tfrac{2}{4} = 1$$

46. $\cot\theta \sec\theta$

$$\frac{\cos\theta}{\sin\theta}\left(\frac{1}{\cos\theta}\right)$$
$$\frac{1}{\sin\theta}$$
$$\csc\theta$$

47. $\csc\theta(\sin\theta - \tan\theta)$

$$\csc\theta\sin\theta - \csc\theta\tan\theta$$
$$\frac{1}{\sin\theta}\sin\theta - \frac{1}{\sin\theta}\frac{\sin\theta}{\cos\theta}$$
$$1 - \frac{1}{\cos\theta}$$
$$1 - \sec\theta$$

48. $\frac{\sin\theta - 1}{\sin\theta}$

$$\frac{\sin\theta}{\sin\theta} - \frac{1}{\sin\theta}$$
$$1 - \csc\theta$$

49. $\frac{\cos\alpha + 2 - \cot\alpha}{\cos\alpha}$

$$\frac{\cos\alpha}{\cos\alpha} + \frac{2}{\cos\alpha} - \frac{\cot\alpha}{\cos\alpha}$$
$$1 + 2\left(\frac{1}{\cos\alpha}\right) - \frac{\frac{\cos\alpha}{\sin\alpha}}{\cos\alpha}$$
$$1 + 2\sec\alpha - \frac{1}{\sin\alpha}$$
$$1 + 2\sec\alpha - \csc\alpha$$

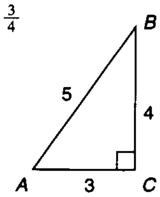
50. $(1 + \sin\theta)(1 - \sin\theta)$

$$1 - \sin\theta + \sin\theta - \sin^2\theta$$
$$1 - \sin^2\theta$$
$$\cos^2\theta + \sin^2\theta - \sin^2\theta$$
$$\cos^2\theta$$

51. 20.9°

Chapter 1 test

1. 26.462° **2.** 64.21° **3.** $6\sqrt{3}$
4. 73.8 ft
5. $\frac{3}{4}$



6. 0.4305 **7.** 4.9313 **8.** 1.6107
9. 603.6 watts **10.** 7.6° **11.** 76.0°
12. $b \approx 9.7$, $c \approx 10.4$, $B = 68.6°$
13. $c \approx 9.5$, $A \approx 33.4°$, $B \approx 56.6°$
14. 15,540 ft
15. $\cos\theta(\sec\theta - \cos\theta)$

$$\cos\theta\sec\theta - \cos^2\theta$$
$$\cos\theta\frac{1}{\cos\theta} - \cos^2\theta$$
$$1 - \cos^2\theta$$
$$\sin^2\theta$$

16. 15.7°

Chapter 2

Exercise 2-1

Answers to odd-numbered problems

1. a. yes **b.** $D = \{3, 4, 6, 7\}$, $R = \{5, 9, 10\}$ **c.** not one to one
3. a. yes
b. $D = \{-2, 3, 4\}$, $R = \{-2, 3, 4\}$
c. $h^{-1} = \{(-2,-2), (4,3), (3,4)\}$
5. not a function **7. a.** yes
b. $D = \{1, 2, 3, 4\}$, $R = \{1, 2, 3, 4\}$
c. $f^{-1} = \{(1,1), (2,2), (3,3), (4,4)\}$
9. a. 9 **b.** 19 **c.** 11 **d.** not defined
e. not defined **11. a.** $(-2,-13)$
b. $(0,-3)$ **c.** $(\sqrt{3}, 5\sqrt{3} - 3)$
d. $(\tfrac{1}{2}, -\tfrac{1}{2})$ **e.** (5,22) **13. a.** $(-2,0)$
b. $(0,-6)$ **c.** $(\sqrt{3}, -\sqrt{3} - 3)$
d. $\left(\frac{1}{2}, \frac{-25}{4}\right)$ **e.** (5,14)
15. a. $(-2,1)$ **b.** $(0,-1)$
c. $(\sqrt{3}, 2 + \sqrt{3})$ **d.** $(\tfrac{1}{2}, -\tfrac{1}{4})$
e. (5,29) **17. a.** $(-2,46)$ **b.** (0,2)
c. $(\sqrt{3}, 26)$ **d.** $(\tfrac{1}{2}, \tfrac{31}{16})$ **e.** (5, 1,852)
19. a. $(-2,-2)$ **b.** (0,0)
c. $\left(\sqrt{3}, \frac{\sqrt{3} - 1}{2}\right)$ **d.** $(\tfrac{1}{2}, \tfrac{1}{7})$ **e.** $(5, \tfrac{5}{8})$
21. a. 494° C **b.** 476° C **c.** 292° C

Solutions to trial exercise problems

8. $g = \{(-3,5), (5,8), (8,13), (13,21)\}$
a. Is a function because no first element repeats.
b. $\text{Domain}_g = \{-3, 5, 8, 13\}$; $\text{Range}_g = \{5, 8, 13, 21\}$
c. Is a one-to-one function because no second element repeats. The inverse is $\{(5,-3), (8,5), (13,8), (21,13)\}$.

19. $h(x) = \dfrac{x}{x+3}$

a. $h(-2) = \dfrac{-2}{-2+3} = \dfrac{-2}{1} = -2; (-2,-2)$

b. $h(0) = \dfrac{0}{0+3} = \dfrac{0}{3} = 0; (0,0)$

c. $h(\sqrt{3}) = \dfrac{\sqrt{3}}{\sqrt{3}+3} = \dfrac{\sqrt{3}}{\sqrt{3}+3} \cdot \dfrac{\sqrt{3}-3}{\sqrt{3}-3} = \dfrac{3-3\sqrt{3}}{3-9}$
$= \dfrac{-3(-1+\sqrt{3})}{-6} = \dfrac{\sqrt{3}-1}{2}; \left(\sqrt{3}, \dfrac{\sqrt{3}-1}{2}\right)$

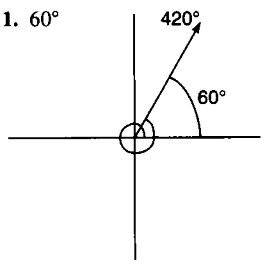
d. $h(\frac{1}{2}) = \dfrac{\frac{1}{2}}{\frac{1}{2}+3} = \dfrac{\frac{1}{2}}{\frac{7}{2}} = \frac{1}{2} \cdot \frac{2}{7} = \frac{1}{7}; (\frac{1}{2}, \frac{1}{7})$

e. $h(5) = \dfrac{5}{5+3} = \dfrac{5}{8}; (5, \frac{5}{8})$

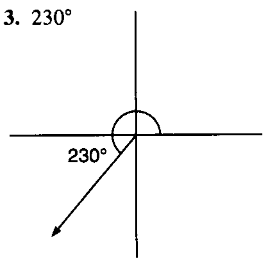
Exercise 2–2

Answers to odd-numbered problems

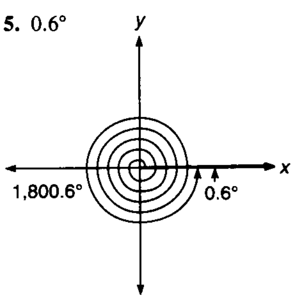
1. 60°



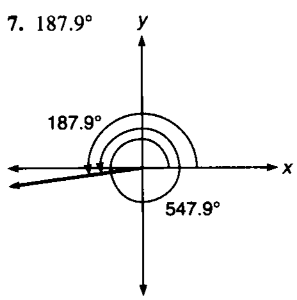
3. 230°



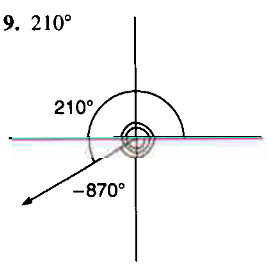
5. 0.6°



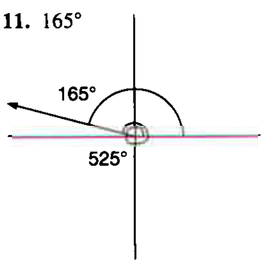
7. 187.9°



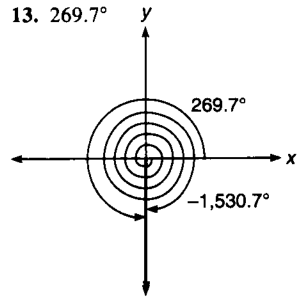
9. 210°



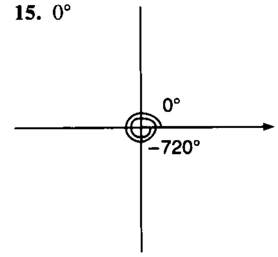
11. 165°



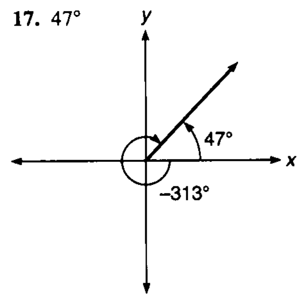
13. 269.7°



15. 0°



17. 47°



19. 347° **21.** 353.9°

23. $\sin\theta = \dfrac{2\sqrt{5}}{5}$, $\cos\theta = \dfrac{\sqrt{5}}{5}$, $\tan\theta = 2$, $\csc\theta = \dfrac{\sqrt{5}}{2}$, $\sec\theta = \sqrt{5}$, $\cot\theta = \dfrac{1}{2}$ **25.** $\sin\theta = \dfrac{8\sqrt{89}}{89}$, $\cos\theta = \dfrac{-5\sqrt{89}}{89}$, $\tan\theta = -\dfrac{8}{5}$, $\csc\theta = \dfrac{\sqrt{89}}{8}$, $\sec\theta = -\dfrac{\sqrt{89}}{5}$, $\cot\theta = \dfrac{-5}{8}$

27. $\sin\theta = \dfrac{-\sqrt{2}}{2}$, $\cos\theta = \dfrac{\sqrt{2}}{2}$, $\tan\theta = -1$, $\csc\theta = -\sqrt{2}$, $\sec\theta = \sqrt{2}$, $\cot\theta = -1$ **29.** $\sin\theta = \dfrac{4\sqrt{17}}{17}$, $\cos\theta = -\dfrac{\sqrt{17}}{17}$, $\tan\theta = -4$, $\csc\theta = \dfrac{\sqrt{17}}{4}$, $\sec\theta = -\sqrt{17}$, $\cot\theta = -\dfrac{1}{4}$

31. $\sin\theta = \dfrac{-3\sqrt{13}}{13}$, $\cos\theta = \dfrac{-2\sqrt{13}}{13}$, $\tan\theta = \dfrac{3}{2}$, $\csc\theta = \dfrac{-\sqrt{13}}{3}$, $\sec\theta = \dfrac{-\sqrt{13}}{2}$, $\cot\theta = \dfrac{2}{3}$

33. $\sin\theta = \dfrac{3\sqrt{38}}{19}$, $\cos\theta = \dfrac{-\sqrt{19}}{19}$, $\tan\theta = -3\sqrt{2}$, $\csc\theta = \dfrac{\sqrt{38}}{6}$, $\sec\theta = -\sqrt{19}$, $\cot\theta = \dfrac{-\sqrt{2}}{6}$ **35.** $\sin\theta = -\dfrac{\sqrt{10}}{5}$, $\cos\theta = -\dfrac{\sqrt{15}}{5}$, $\tan\theta = \dfrac{\sqrt{6}}{3}$, $\csc\theta = -\dfrac{\sqrt{10}}{2}$, $\sec\theta = -\dfrac{\sqrt{15}}{3}$, $\cot\theta = \dfrac{\sqrt{6}}{2}$ **37.** $\sin\theta = \dfrac{-\sqrt{10}}{4}$, $\cos\theta = \dfrac{\sqrt{6}}{4}$, $\tan\theta = \dfrac{-\sqrt{15}}{3}$, $\csc\theta = \dfrac{-2\sqrt{10}}{5}$, $\sec\theta = \dfrac{2\sqrt{6}}{3}$, $\cot\theta = \dfrac{-\sqrt{15}}{5}$

39. $\sin\theta = -\dfrac{\sqrt{5}}{5}$, $\cos\theta = \dfrac{2\sqrt{5}}{5}$, $\tan\theta = -\dfrac{1}{2}$, $\csc\theta = -\sqrt{5}$, $\sec\theta = \dfrac{\sqrt{5}}{2}$, $\cot\theta = -2$ **41.** $\sin\theta = \dfrac{\sqrt{3}}{3}$, $\cos\theta = \dfrac{\sqrt{6}}{3}$, $\tan\theta = \dfrac{\sqrt{2}}{2}$, $\csc\theta = \sqrt{3}$, $\sec\theta = \dfrac{\sqrt{6}}{2}$, $\cot\theta = \sqrt{2}$

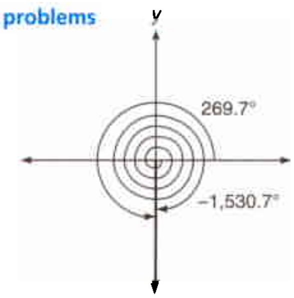
43. $\sin\theta = \dfrac{2\sqrt{5}}{5}$, $\cos\theta = \dfrac{\sqrt{5}}{5}$, $\tan\theta = 2$, $\csc\theta = \dfrac{\sqrt{5}}{2}$, $\sec\theta = \sqrt{5}$, $\cot\theta = \dfrac{1}{2}$ **45.** $\dfrac{1}{\cos\theta} = \dfrac{1}{\frac{x}{r}} = \dfrac{r}{x} = \sec\theta$

47. $\dfrac{\cos\theta}{\sin\theta} = \dfrac{\frac{x}{r}}{\frac{y}{r}} = \dfrac{x}{y} = \cot\theta$ **49.** $\dfrac{1}{\csc\theta} = \dfrac{1}{\frac{r}{y}} = \dfrac{y}{r} = \sin\theta$

51. (x_1,y_1), (x_2,y_2) represent two points with the same trigonometric functions. The tangent for the point (x_1,y_1) is $\dfrac{y_1}{x_1}$. The tangent for the point (x_2,y_2) is $\dfrac{y_2}{x_2}$, but $\dfrac{y_1}{x_1} = \dfrac{y_2}{x_2}$, since all trigonometric functions are the same. Also, $\dfrac{y_1}{x_1}$ is the slope of the line through $(0,0)$ and (x_1,y_1). Likewise for $\dfrac{y_2}{x_2}$. So $\dfrac{y_1}{x_1} = \dfrac{y_2}{x_2} = m$. Thus, both (x_1,y_1) and (x_2,y_2) lie on the line $y = mx$. They must be on the same terminal side, since all the trigonometric functions have the same sign.

Solutions to trial exercise problems

13. $-1{,}530.3°$
$-1{,}530 \div 360 = -4.25$;
$-1{,}530.3° + 5(360°) = 269.7°$
(See the diagram.)



35. $(-\sqrt{3}, -\sqrt{2})$

$$r = \sqrt{(-\sqrt{3})^2 + (-\sqrt{2})^2} = \sqrt{3+2} = \sqrt{5}$$

$$\sin\theta = \frac{y}{r} = \frac{-\sqrt{2}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = -\frac{\sqrt{10}}{5};$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{5}}{-\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{10}}{2};$$

$$\cos\theta = \frac{x}{r} = \frac{-\sqrt{3}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = -\frac{\sqrt{15}}{5};$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{5}}{-\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{15}}{3};$$

$$\tan\theta = \frac{y}{x} = \frac{-\sqrt{2}}{-\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3};$$

$$\cot\theta = \frac{x}{y} = \frac{-\sqrt{3}}{-\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$$

38. $(b,-2b) = (x,y)$

$$r = \sqrt{b^2 + (-2b)^2} = \sqrt{5b^2} = \sqrt{5}\sqrt{b^2} = \sqrt{5}b$$

$$\sin\theta = \frac{y}{r} = \frac{-2b}{\sqrt{5}b} = -\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$\cos\theta = \frac{x}{r} = \frac{b}{\sqrt{5}b} = \frac{1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{-2b}{b} = -2$$

$$\csc\theta = \frac{1}{\sin\theta} = -\frac{\sqrt{5}}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \sqrt{5}$$

$$\cot\theta = \frac{1}{\tan\theta} = -\frac{1}{2}$$

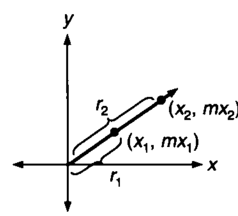
50. As explained in the problem and shown in the figure we have two points (x_1,mx_1) and (x_2,mx_2) on the terminal side of an angle. Note that the value m in each case is the same number, since it is the slope of the line on which both points lie. Using the first point (x_1,mx_1),

$$r_1 = \sqrt{(x_1)^2 + (mx_1)^2} = \sqrt{x_1^2 + m^2x_1^2}$$
$$= \sqrt{x_1^2(1+m^2)} = |x_1|\sqrt{1+m^2}$$

$$\sin\theta = \frac{y}{r_1} = \frac{mx_1}{|x_1|\sqrt{1+m^2}} = \frac{x_1}{|x_1|}\cdot\frac{m}{\sqrt{1+m^2}}$$

The value of $\dfrac{x_1}{|x_1|}$ is 1 or -1, depending on the sign of x_1. Thus,

$$\sin\theta = \begin{cases} \dfrac{m}{\sqrt{1+m^2}} & \text{if } x_1 > 0 \\ -\dfrac{m}{\sqrt{1+m^2}} & \text{if } x_1 < 0 \end{cases}$$



Using the second point (x_2,mx_2),

$$r_2 = \sqrt{(x_2)^2 + (mx_2)^2} = \sqrt{x_2^2 + m^2x_2^2}$$
$$= \sqrt{x_2^2(1+m^2)} = |x_2|\sqrt{1+m^2}$$

$$\sin\theta = \frac{y}{r_2} = \frac{mx_2}{|x_2|\sqrt{1+m^2}} = \frac{x_2}{|x_2|}\cdot\frac{m}{\sqrt{1+m^2}}$$

As above, the value of $\dfrac{x_2}{|x_2|}$ is 1 or -1, depending on the sign of x_2. Thus,

$$\sin\theta = \begin{cases} \dfrac{m}{\sqrt{1+m^2}} & \text{if } x_2 > 0 \\ -\dfrac{m}{\sqrt{1+m^2}} & \text{if } x_2 < 0 \end{cases}$$

Since x_1 and x_2 lie in the same quadrant, they have the same sign. Therefore, we obtain the same value for $\sin\theta$ using either point.

Exercise 2-3

Answers to odd-numbered problems

1. II **3.** I **5.** IV **7.** II **9.** IV
11. III **13.** 15.8° **15.** 67.1°
17. 75.3° **19.** 49.3° **21.** 1°
23. 80.5° **25.** 72° **27.** $\frac{\sqrt{2}}{2}$ **29.** $-\frac{1}{2}$
31. $-\sqrt{3}$ **33.** $-\frac{\sqrt{3}}{2}$ **35.** $\frac{1}{2}$
37. $-\frac{\sqrt{3}}{3}$ **39.** 0 **41.** 1 **43.** $\frac{1}{2}$
45. $-\frac{2\sqrt{3}}{3}$ **47.** 0.9178 **49.** 0.6899
51. 0.7813 **53.** 1.0263 **55.** 0.9967
57. -1.0367 **59.** -0.6845
61. 14.5° **63.** 120° **65.** $-58.0°$
67. $-36.2°$ **69. a.** 110.31 volts
b. 156 volts **c.** 89.48 volts
d. -65.93 volts **e.** 132.73 volts
f. 0 volts **71. a.** 200 lb
b. 181.3 lb **c.** 128.6 lb

73. $\sin 60° \stackrel{?}{=} 2 \sin 30°$

$$\frac{\sqrt{3}}{2} \stackrel{?}{=} 2 \cdot \frac{1}{2}$$

$\frac{\sqrt{3}}{2} \neq 1$; No, it is false.

75. Use the values 30°, 60°, 90° to see if the statement $\sin(\alpha + \beta) = \sin \alpha + \sin \beta$ is true.

$$\sin(30° + 60°) \stackrel{?}{=} \sin 30° + \sin 60°$$

$$\sin 90° \stackrel{?}{=} \sin 30° + \sin 60°$$

$$1 \stackrel{?}{=} \frac{1}{2} + \frac{\sqrt{3}}{2}$$

$1 \neq \frac{1 + \sqrt{3}}{2}$; No, it is false.

Solutions to trial exercise problems

6. $\sec \theta > 0$, $\csc \theta < 0$
If $\sec \theta > 0$, then $\cos \theta > 0$, so θ is in quadrant I or quadrant IV. If $\csc \theta < 0$, then $\sin \theta < 0$, so θ is in quadrant III or quadrant IV. Therefore, θ is in quadrant IV.

19. 130.7°
Since $90° < 130.7° < 180°$, this angle terminates in quadrant II. In quadrant II, $\theta' = 180° - \theta = 180° - 130.7° = 49.3°$

43. $\sin(-690°)$
$-690°$ is coterminal with 30° (add $-690° + 2[360°]$), so $\sin(-690°) = \sin 30° = \frac{1}{2}$

52. $\tan 527.2°$
$527.2° - 360° = 167.2°$, so $\tan 527.2° = \tan 167.2°$. Since 167.2° terminates in quadrant II, $\theta' = 180° - 167.2° = 12.8°$. The tangent function is negative in quadrant II, so we know that
$\tan 167.2° = -\tan 12.8°$
$= -0.2272$ (to four decimal places).

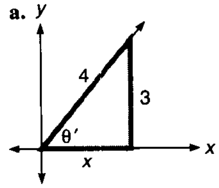
65. $\tan \theta = -\frac{8}{5}$
$\theta = \tan^{-1}(-\frac{8}{5}) \approx -58.0°$
Display: $\boxed{-57.99461679}$

Exercise 2-4

Answers to odd-numbered problems

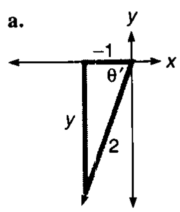
1. 55.6° **3.** 33.3° **5.** 200.0°
7. 358.0° **9.** 22.0° **11.** 168.7°
13. 224.4°

15. a.



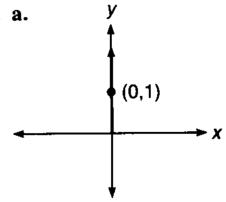
b. $\csc \theta = \frac{4}{3}$, $\cos \theta = \frac{\sqrt{7}}{4}$, $\sec \theta = \frac{4\sqrt{7}}{7}$, $\tan \theta = \frac{3\sqrt{7}}{7}$, $\cot \theta = \frac{\sqrt{7}}{3}$ **c.** $\theta = \theta' \approx 48.6°$

17. a.



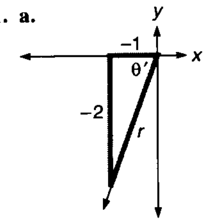
b. $\sin \theta = \frac{-\sqrt{3}}{2}$, $\csc \theta = \frac{-2\sqrt{3}}{3}$, $\sec \theta = -2$, $\tan \theta = \sqrt{3}$, $\cot \theta = \frac{\sqrt{3}}{3}$
c. $\theta' = 60°$, $\theta = 240°$

19. a.



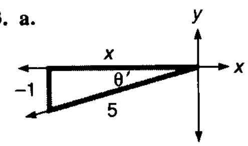
b. $\cos \theta = 0$, $\tan \theta$ undefined, $\cot \theta = 0$, $\csc \theta = 1$, $\sec \theta$ undefined
c. $\theta = \theta' = 90°$

21. a.



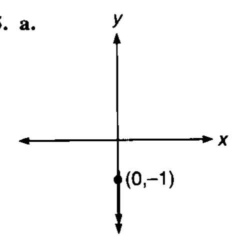
b. $\cot \theta = \frac{1}{2}$, $\sin \theta = \frac{-2\sqrt{5}}{5}$, $\cos \theta = \frac{-\sqrt{5}}{5}$, $\csc \theta = \frac{-\sqrt{5}}{2}$, $\sec \theta = -\sqrt{5}$
c. $\theta' \approx 63.4°$, $\theta \approx 243.4°$

23. a.



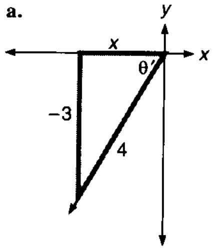
b. $\sin \theta = -\frac{1}{5}$, $\cos \theta = \frac{-2\sqrt{6}}{5}$, $\sec \theta = \frac{-5\sqrt{6}}{12}$, $\tan \theta = \frac{\sqrt{6}}{12}$, $\cot \theta = 2\sqrt{6}$
c. $\theta' \approx 11.5°$, $\theta \approx 191.5°$

25. a.



b. $\sin \theta = -1$, $\cos \theta = 0$, $\sec \theta$ undefined, $\tan \theta$ undefined, $\cot \theta = 0$ **c.** $\theta' = 90°$, $\theta = 270°$

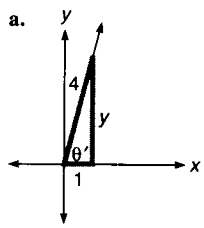
27. a.



b. $\csc\theta = -\frac{4}{3}$, $\cos\theta = \frac{-\sqrt{7}}{4}$, $\sec\theta = \frac{-4\sqrt{7}}{7}$, $\tan\theta = \frac{3\sqrt{7}}{7}$, $\cot\theta = \frac{\sqrt{7}}{3}$

c. $\theta' \approx 48.6°$, $\theta \approx 228.6°$

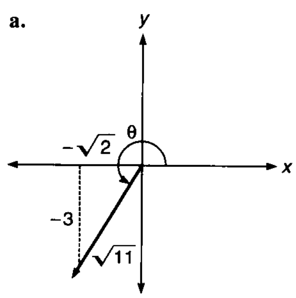
29. a.



b. $\cos\theta = \frac{1}{4}$, $\sin\theta = \frac{\sqrt{15}}{4}$, $\csc\theta = \frac{4\sqrt{15}}{15}$, $\tan\theta = \sqrt{15}$, $\cot\theta = \frac{\sqrt{15}}{15}$

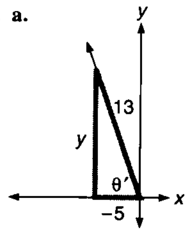
c. $\theta = \theta' \approx 75.5°$

31. a.



b. $\tan\theta = \frac{3\sqrt{2}}{2}$, $\cos\theta = -\frac{\sqrt{22}}{11}$, $\sin\theta = -\frac{3\sqrt{11}}{11}$, $\csc\theta = -\frac{\sqrt{11}}{3}$, $\sec\theta = -\frac{\sqrt{22}}{2}$ **c.** $\theta' \approx 64.8°$, $\theta \approx 244.8°$

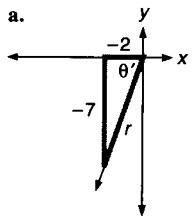
33. a.



b. $\sec\theta = \frac{-13}{5}$, $\sin\theta = \frac{12}{13}$, $\csc\theta = \frac{13}{12}$, $\tan\theta = \frac{-12}{5}$, $\cot\theta = \frac{-5}{12}$

c. $\theta' \approx 67.4°$, $\theta \approx 112.6°$

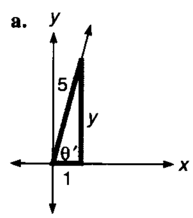
35. a.



b. $\cot\theta = \frac{2}{7}$, $\sin\theta = \frac{-7\sqrt{53}}{53}$, $\csc\theta = \frac{-\sqrt{53}}{7}$, $\cos\theta = \frac{-2\sqrt{53}}{53}$, $\sec\theta = \frac{-\sqrt{53}}{2}$

c. $\theta \approx 74.1°$, $\theta \approx 254.1°$

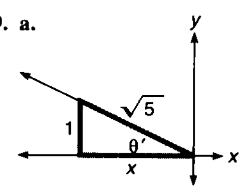
37. a.



b. $\cos\theta = \frac{1}{5}$, $\sin\theta = \frac{2\sqrt{6}}{5}$, $\csc\theta = \frac{5\sqrt{6}}{12}$, $\tan\theta = 2\sqrt{6}$, $\cot\theta = \frac{\sqrt{6}}{12}$

c. $\theta = \theta' \approx 78.5°$

39. a.



b. $\csc\theta = \sqrt{5}$, $\cos\theta = \frac{-2\sqrt{5}}{5}$, $\sec\theta = \frac{-\sqrt{5}}{2}$, $\cot\theta = -2$, $\tan\theta = -\frac{1}{2}$

c. $\theta' \approx 26.6°$, $\theta \approx 153.4°$

41. a. $\sin\theta = \frac{-5\sqrt{29}}{29}$, $\cos\theta = \frac{2\sqrt{29}}{29}$, $\tan\theta = -\frac{5}{2}$, $\sec\theta = \frac{\sqrt{29}}{2}$, $\csc\theta = \frac{-\sqrt{29}}{5}$, $\cot\theta = -\frac{2}{5}$ **b.** 291.8°

43. (−4.85 mm,4.77 mm)

45. (−5.77 cm,−5.89 cm)

47. 8.8 cm, 60.6°; −8.8 cm, 119.4°; −8.8 cm, 240.6°; 8.8 cm, 299.4°

49. $y \approx -13.5$ in., $x \approx -22.0$ in.

51. $\cot\theta = \frac{1}{u}$, $\sin\theta = \frac{u}{\sqrt{u^2+1}}$, $\csc\theta = \frac{\sqrt{u^2+1}}{u}$, $\cos\theta = \frac{1}{\sqrt{u^2+1}}$, $\sec\theta = \sqrt{u^2+1}$ **53.** $\cot\theta = \frac{1}{u}$, $\sin\theta = \frac{-u}{\sqrt{u^2+1}}$, $\csc\theta = \frac{-\sqrt{u^2+1}}{u}$, $\cos\theta = \frac{-1}{\sqrt{u^2+1}}$, $\sec\theta = -\sqrt{u^2+1}$

55. $\sec\theta = \frac{1}{1-u}$, $\sin\theta = \sqrt{2u-u^2}$, $\csc\theta = \frac{1}{\sqrt{2u-u^2}}$, $\tan\theta = \frac{\sqrt{2u-u^2}}{1-u}$, $\cot\theta = \frac{1-u}{\sqrt{2u-u^2}}$ **57.** 663.4 ft

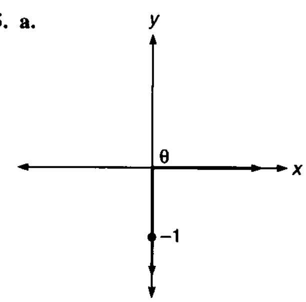
Solutions to trial exercise problems

13. $\cos\theta = -\frac{5}{7}$, $\tan\theta > 0$

$\theta' = \cos^{-1}\frac{5}{7} \approx 44.4°$

$\cos\theta < 0$, $\tan\theta > 0$, so θ terminates in quadrant III. Therefore, $\theta = 180° + \theta' \approx 180° + 44.4° \approx 224.4°$.

25. a.



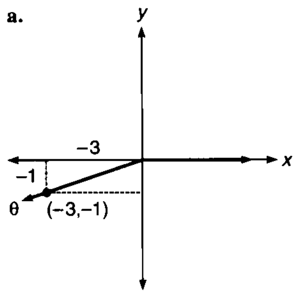
b. $\csc\theta = -1$

$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-1} = -1$; $\sin\theta = \frac{y}{r} = -1 = \frac{-1}{1}$, so we see that $y = -1$ and $r = 1$ would work. This means

that x must be 0, since $x^2 + y^2 = r^2$. Thus, the point $(0, -1)$ is on the angle, which is shown in the diagram. $\cos\theta = \frac{0}{1} = 0$; $\sec\theta = \frac{1}{\cos\theta}$, which is not defined in this case. $\tan\theta = \frac{y}{x} = \frac{-1}{0}$ (not defined); $\cot\theta = \frac{x}{y} = \frac{0}{-1} = 0$.

c. We can see that $\theta = 270°$ from the diagram.

34. a.



b. $\cos\theta = -\frac{3}{\sqrt{10}}$, $\sin\theta < 0$; $x = -3$, $r = \sqrt{10}$.

Computing, $x^2 + y^2 = r^2$

$$9 + y^2 = 10$$
$$y = \pm 1$$

Since $\cos\theta < 0$ and $\sin\theta < 0$, θ terminates in quadrant III, so $y = -1$. (See the diagram.)

$\sin\theta = \frac{y}{r} = \frac{-1}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{-\sqrt{10}}{10}$; $\tan\theta = \frac{y}{x} = \frac{-1}{-3} = \frac{1}{3}$; $\csc\theta = \frac{1}{\sin\theta} = -\sqrt{10}$; $\sec\theta = \frac{1}{\cos\theta} = \frac{-\sqrt{10}}{3}$; $\cot\theta = \frac{1}{\tan\theta} = 3$.

c. $\theta = 180° + \tan^{-1}\frac{1}{3} \approx 180° + 18.4° \approx 198.4°$

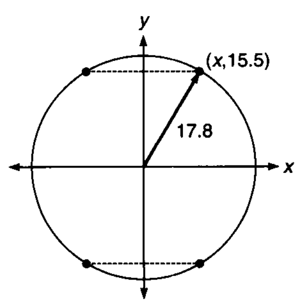
47. $\sin\theta' = \frac{y}{r} = \frac{15.5}{17.8}$ (See diagram.)

$\theta' = 60.6°$. The four angles are

$\theta = \theta' = 60.6°$
$= 180° - 60.6° = 119.4°$
$= 180° + 60.6° = 240.6°$
$= 360° - 60.6° = 299.4°$

We know that $r^2 = x^2 + y^2$, so

$17.8^2 = x^2 + 15.5^2$
$17.8^2 - 15.5^2 = x^2$
$76.59 = x^2$, $8.75 = x$. Thus, $x = \pm 8.8$.

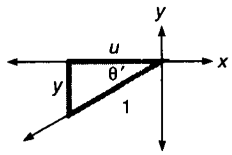


49. 4 ft 3.5 in. = 51.5 in.; $r = \frac{51.5}{2}$ in. = 25.75 in.

$x = r\cos\theta = 25.75\cos 211.5° = -22.0$ in.

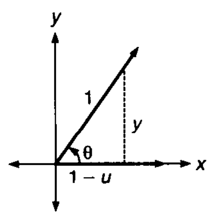
$y = r\sin\theta = 25.75\sin 211.5° = -13.5$ in.

52. $\cos\theta = u$, and θ terminates in quadrant III. (See the diagram.)



Using the Pythagorean theorem, we find $y = -\sqrt{1 - u^2}$. $\sin\theta = \frac{y}{r} = -\sqrt{1 - u^2}$, $\tan\theta = \frac{y}{x} = \frac{-\sqrt{1 - u^2}}{u}$, $\csc\theta = \frac{1}{\sin\theta} = \frac{-1}{\sqrt{1 - u^2}}$, $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{u}$, $\cot\theta = \frac{1}{\tan\theta} = -\frac{u}{\sqrt{1 - u^2}}$

55. $\cos\theta = 1 - u$, θ in quadrant I. (See the diagram.)



$1^2 = y^2 + (1 - u)^2$
$1 - (1 - 2u + u^2) = y^2$
$2u - u^2 = y^2$
$\sqrt{2u - u^2} = y \ (y > 0)$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{1 - u}$;
$\sin\theta = \frac{y}{r} = \sqrt{2u - u^2}$;
$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\sqrt{2u - u^2}}$;
$\tan\theta = \frac{y}{x} = \frac{\sqrt{2u - u^2}}{1 - u}$;
$\cot\theta = \frac{1}{\tan\theta} = \frac{1 - u}{\sqrt{2u - u^2}}$

57. $AB = 512.4$ feet, $AP = 322.6$ feet, $b = 28.3°$.

$\sin p = \frac{AB\sin b}{AP} = \frac{512.4(\sin 28.3°)}{322.6} \approx 0.7530$

$p \approx 48.85°$

$a = 180° - (b + p) \approx 180° - (28.3° + 48.85°) \approx 102.85°$

$BP = \frac{AP\sin a}{\sin b} \approx \frac{322.6(\sin 102.85°)}{\sin 28.3°} \approx 663.4$ feet

Exercise 2–5

Answers to odd-numbered problems

1. $x^2 + y^2 = 1$ **3.** $\frac{\pi}{4}$, 0.79

5. $\frac{5\pi}{9}$, 1.75 **7.** $\frac{-5\pi}{3}$, −5.24

9. $\frac{3\pi}{2}$, 4.71 **11.** $\frac{127\pi}{180}$, 2.22

13. $\frac{-61\pi}{36}$, −5.32 **15.** 330°

17. 108° **19.** 40° **21.** −510°

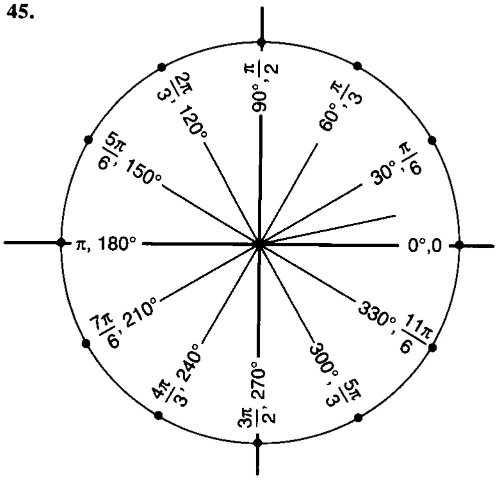
23. $\frac{270°}{\pi}$, 85.94° **25.** $\frac{-2{,}160°}{17\pi}$, −40.44°

27. $\frac{360°}{\pi}$, 114.59° **29.** $\frac{-900°}{\pi}$, −286.48°

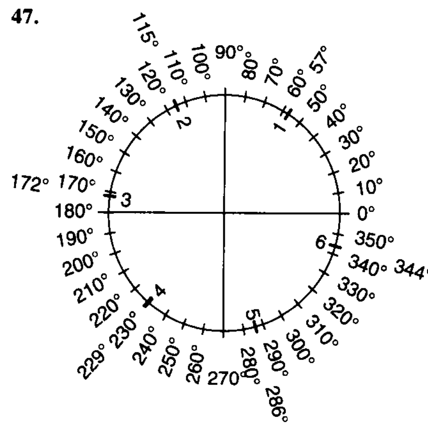
31. $\frac{\pi}{3}$ **33.** $\frac{\pi}{6}$ **35.** $\frac{\pi}{3}$ **37.** $\frac{\pi}{4}$

39. $\frac{\pi}{3}$ **41.** $\frac{\pi}{3}$ **43.** $\frac{\pi}{6}$

45.



47.



49. 2.7 **51.** 6.5 inches **53.** 24 inches **55.** 98 in.^{2} **57.** 33.93 cm^2 **59.** 7.5 in.^{2} **61.** 10.60 mm^2 **63.** 1.24 **65.** 35.5 gallons **67.** 30 mm

Solutions to trial exercise problems

12. $-422°$

$\frac{s}{\pi} = \frac{\theta°}{180°}$, so $\frac{s}{\pi} = \frac{-422°}{180°}$, (Cross multiply.) $s \cdot 180° = \pi(-422°)$, $s = \frac{\pi(-422°)}{180°}$, $s = \frac{-211\pi}{90}$ or -7.37

21. $-\frac{17\pi}{6}$

$\frac{s}{\pi} = \frac{\theta°}{180°}$, so $\frac{\frac{-17\pi}{6}}{\pi} = \frac{\theta°}{180°}$, (Cross multiply.) $\frac{-17\pi(180°)}{6} = \pi\theta°$,

$-17\pi(30°) = \pi\theta°$, $\frac{-17\pi(30°)}{\pi} = \theta°$,

$-17(30°) = \theta°$, $-510° = \theta°$

29. -5

$\frac{s}{\pi} = \frac{\theta°}{180°}$, so $\frac{-5}{\pi} = \frac{\theta°}{180°}$, (Cross multiply.) $-5(180°) = \pi\theta°$, $\frac{-5(180°)}{\pi}$

$= \theta°$, $\frac{-900°}{\pi} = \theta°$, $-286.48° = \theta°$

43. $-\frac{7\pi}{6}$

$-\frac{7\pi}{6} + \pi = -\frac{7\pi}{6} + \frac{1\pi}{6} = \frac{5\pi}{6}$

Thus, $\frac{5\pi}{6}$ and $-\frac{7\pi}{6}$ are coterminal (and therefore have the same reference angle). Now find in which quadrant $\frac{5\pi}{6}$ terminates.

$0 \quad \frac{\pi}{2} \quad \pi$

$0 \quad \frac{3\pi}{6} \quad \frac{6\pi}{6}$

$\frac{3\pi}{6} < \frac{5\pi}{6} < \frac{6\pi}{6}$, so $\frac{\pi}{2} < \frac{5\pi}{6} < \pi$, so $\frac{5\pi}{6}$ terminates in quadrant II. Therefore, θ' $= \pi - \theta = \frac{6\pi}{6} - \frac{5\pi}{6} = \frac{\pi}{6}$.

52. $s = \frac{L}{r}$; $L = 14.5$ mm, diameter $= 10.3$ mm. $r = \frac{10.3\text{ mm}}{2} = 5.15$ mm,

$s = \frac{14.5\text{ mm}}{5.15\text{ mm}} = \frac{1{,}450}{515} = \frac{290}{103} \approx 2.8155$

$\frac{s}{\pi} = \frac{\theta°}{180°}$, so $\frac{2.8155}{\pi} \approx \frac{\theta°}{180°}$, (Cross multiply.) $2.8155(180°) \approx \pi\theta°$,

$\frac{2.8155(180°)}{\pi} = \theta°$, $161.318° \approx \theta°$

Thus, $\theta° = 161.3°$ or 2.8 (radians).

53. Convert 85° to radians.

$\frac{85°}{180°} = \frac{s}{\pi}$

$s = \frac{85\pi}{180} = \frac{17\pi}{36}$

The radius of the wheel is $32.4'' \div 2 = 16.2''$. The car will move whatever arc length the angle determines on its circumference: $L = rs$

$L = 16.2'' \cdot \frac{17\pi}{36} = 7.65\pi \approx 24.0$ inches, or 2 feet

61. 15°, 9 mm

15° is $\frac{\pi}{12}$ radians

$A = \frac{1}{2}sr^2 = \frac{1}{2} \cdot \frac{\pi}{12} \cdot 9^2 = \frac{81\pi}{24} = \frac{27\pi}{8}$ ≈ 10.60 mm^2

Exercise 2-6

Answers to odd-numbered problems

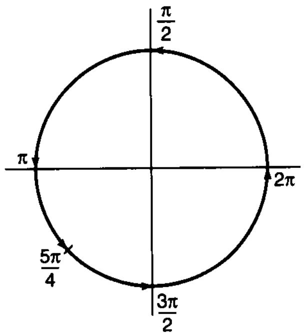
1. $\frac{\sqrt{3}}{2}$ **3.** $\frac{\sqrt{3}}{2}$ **5.** $-\frac{1}{2}$ **7.** $\frac{\sqrt{2}}{2}$ **9.** $-\sqrt{3}$ **11.** $\sqrt{3}$ **13.** $\frac{1}{2}$ **15.** 0.7833 **17.** 0.5463 **19.** 1.5523 **21.** 0.7457 **23.** 1.4235 **25.** 1.6709 **27.** $\frac{4\pi}{3}$ **29.** $\frac{5\pi}{3}$ **31.** $\frac{7\pi}{4}$ **33.** $\frac{11\pi}{6}$ **35.** 1.9513 **37.** 3.6259 **39.** 3.4814 **41.** 0.17 **43.** 1.19 **45.** π **47.** $\frac{\pi}{18}$ **49.** $\frac{\pi}{2}$ or $\frac{\pi}{3}$ **51.** $\frac{\pi}{3}$ or $\frac{\pi}{2}$ **53. a.** -0.03 **b.** -0.37 **55.** 18.18 cm **57. a.** 0.099833417 **b.** 0.479425533 **c.** 0.841468254 **d.** 0.499999992

Solutions to trial exercise problems

2. $\tan \dfrac{5\pi}{4}$

First we must find the quadrant in which $\dfrac{5\pi}{4}$ terminates.

Remember the following correspondence between values in radians and quadrants, as illustrated in the diagram.



0 to $\frac{1}{2}\pi$: quadrant I; $\frac{1}{2}\pi$ to 1π: quadrant II; 1π to $1\frac{1}{2}\pi$: quadrant III; $1\frac{1}{2}\pi$ to 2π: quadrant IV.

$\dfrac{5\pi}{4} = 1\dfrac{1}{4}\pi$, which, therefore, terminates in quadrant III. For a value that corresponds to an angle that terminates in quadrant III, we form the reference angle by subtracting π:

$1\frac{1}{4}\pi - \pi = \frac{1}{4}\pi = \frac{\pi}{4}$. $\tan\frac{\pi}{4} = \tan 45° = 1$ so $\tan\dfrac{5\pi}{4} = \pm 1$.

Since the corresponding angle terminates in quadrant III, where the tangent function is positive, $\tan\dfrac{5\pi}{4} = 1$.

3. $\cos\dfrac{11\pi}{6}$

$\dfrac{11\pi}{6} = 1\dfrac{5}{6}\pi$, which, therefore, represents an angle that terminates in quadrant IV (see discussion of problem 2). In this quadrant we subtract the value from 2π to obtain the reference angle.

$2\pi - 1\dfrac{5}{6}\pi = \dfrac{1}{6}\pi = \dfrac{\pi}{6}$. $\cos\dfrac{\pi}{6} = \cos 30° = \dfrac{\sqrt{3}}{2}$

Since the cosine function is positive in quadrant IV, we know that our result is positive. Thus, $\cos\dfrac{11\pi}{6} = \dfrac{\sqrt{3}}{2}$.

6. $\sin\dfrac{5\pi}{6}$

$\dfrac{5\pi}{6} = \dfrac{5}{6}\pi$, which corresponds to an angle that terminates in quadrant II. Therefore, the reference angle is $\pi - \frac{5}{6}\pi = \frac{1}{6}\pi = \dfrac{\pi}{6}$. $\sin\dfrac{\pi}{6} = \dfrac{1}{2}$, and $\dfrac{5\pi}{6}$ terminates in quadrant II where the sine function is positive, so $\sin\dfrac{5\pi}{6} = \dfrac{1}{2}$.

24. $\sec 5.2 = \dfrac{1}{\cos 5.2} \approx 2.1344$

5.2 $\boxed{\cos}$ $\boxed{1/x}$

$\boxed{\text{TI-81}}$ $\boxed{(}$ $\boxed{\text{COS}}$ 5.2 $\boxed{)}$ $\boxed{x^{-1}}$ $\boxed{\text{ENTER}}$

Display: $\boxed{2.134395767}$

Make sure calculator is in radian mode.

34. $\sin\theta = -0.5624$

$\theta' = \sin^{-1} 0.5624$

Use the positive value.

$\theta' \approx 0.5973$

$\sin\theta < 0$, $\tan\theta > 0$, so θ terminates in quadrant III. Thus, $\theta = \pi + \theta' \approx 3.74$.

48. $2\sin\theta - 1 = 0$ or $\sin\theta - 1 = 0$

Zero product property: If $ab = 0$ then $a = 0$ or $b = 0$.

$2\sin\theta = 1$ or $\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\frac{1}{2} = \dfrac{\pi}{6}$ or $\theta = \sin^{-1}1 = \dfrac{\pi}{2}$

$\theta = \dfrac{\pi}{6}$ or $\dfrac{\pi}{2}$

55. $V = \dfrac{\sqrt{a^2 + b^2 - 2ab\cos\theta}}{\sin\theta}$, $a = 6.2$ cm, $b = 3.5$ cm, $\theta = 2.6$.

$V = \dfrac{\sqrt{6.2^2 + 3.5^2 - 2(6.2)(3.5)\cos 2.6}}{\sin 2.6} \approx 18.18$ cm

To use a calculator, make sure it is in radian angle mode.

Ⓐ 6.2 $\boxed{x^2}$ $\boxed{+}$ 3.5 $\boxed{x^2}$ $\boxed{-}$ 2 $\boxed{\times}$ 6.2 $\boxed{\times}$ 3.5 $\boxed{\times}$ 2.6 $\boxed{\text{COS}}$ $\boxed{=}$ $\boxed{\sqrt{x}}$ $\boxed{\div}$ 2.6 $\boxed{\text{SIN}}$ $\boxed{=}$

Ⓟ 6.2 $\boxed{x^2}$ 3.5 $\boxed{x^2}$ $\boxed{+}$ 2 $\boxed{\text{ENTER}}$ 6.2 $\boxed{\times}$ 3.5 $\boxed{\times}$ 2.6 $\boxed{\text{COS}}$ $\boxed{\times}$ $\boxed{-}$ $\boxed{\sqrt{x}}$ 2.6 $\boxed{\text{SIN}}$ $\boxed{\div}$

$\boxed{\text{TI-81}}$ $\boxed{\sqrt{\ }}$ $\boxed{(}$ 6.2 $\boxed{x^2}$ $\boxed{+}$ 3.5 $\boxed{x^2}$ $\boxed{-}$ 2 $\boxed{\times}$ 6.2 $\boxed{\times}$ 3.5 $\boxed{\times}$ $\boxed{\text{COS}}$ 2.6 $\boxed{)}$ $\boxed{\div}$ $\boxed{\text{SIN}}$ 2.6 $\boxed{\text{ENTER}}$

Display: $\boxed{18.18497291}$

Chapter 2 review

1. a. yes **b.** $D = \{1, 4, 6, 7\}$, $R = \{7, 5, 10\}$ **c.** not one to one

2. a. yes

b. $D = \{-2, -1, 1, 2\}$, $R = \{-3, 1, 2, 5\}$

c. $\{(-3,-2), (1,-1), (2,1), (5,2)\}$

3. Not a function **4. a.** $(-1,6)$

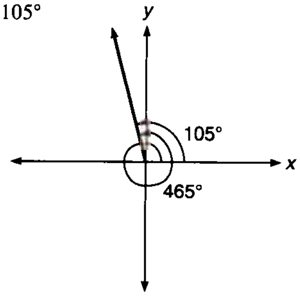
b. $(0,4)$ **c.** $(\sqrt{5}, 4 - 2\sqrt{5})$ **d.** $(\frac{1}{3}, \frac{10}{3})$

5. a. $(-1,8)$ **b.** $(0,3)$

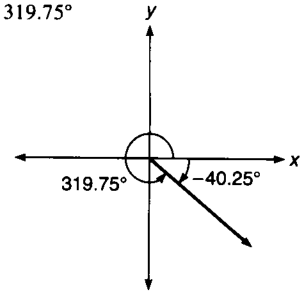
c. $(\sqrt{5}, 18 - 2\sqrt{5})$ **d.** $(\frac{1}{3}, \frac{8}{3})$

6. **a.** $(-1,-4)$ **b.** $(0,-5)$ **c.** $(\sqrt{5},20)$ **d.** $(\frac{1}{3},-4\frac{80}{81})$ **7.** **a.** $(-1,\frac{3}{2})$ **b.** $(0,0)$ **c.** $\left(\sqrt{5},\dfrac{15+3\sqrt{5}}{4}\right)$ **d.** $(\frac{1}{3},-\frac{3}{2})$

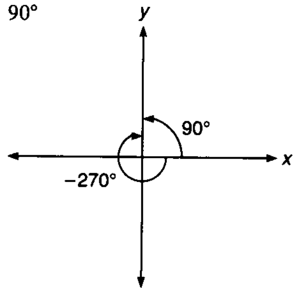
8. 105°



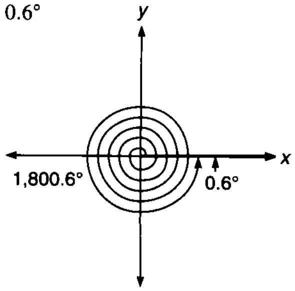
9. 319.75°



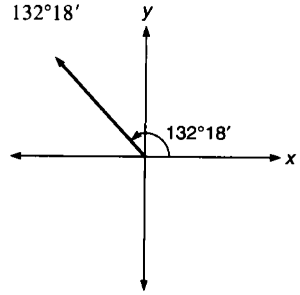
10. 90°



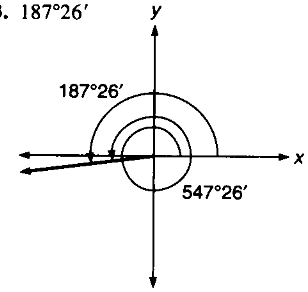
11. 0.6°



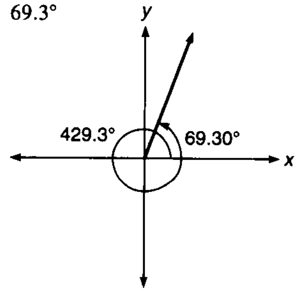
12. 132°18′



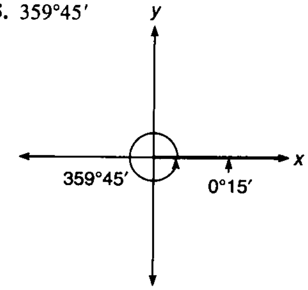
13. 187°26′



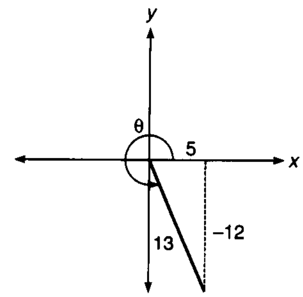
14. 69.3°



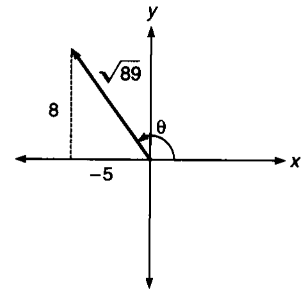
15. 359°45′



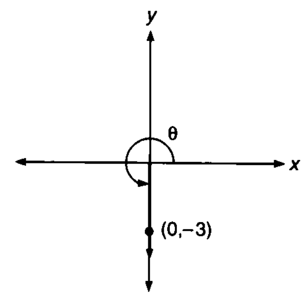
16. $\sin\theta = \frac{-12}{13}$, $\csc\theta = \frac{-13}{12}$, $\cos\theta = \frac{5}{13}$, $\sec\theta = \frac{13}{5}$, $\tan\theta = -\frac{12}{5}$, $\cot\theta = -\frac{5}{12}$



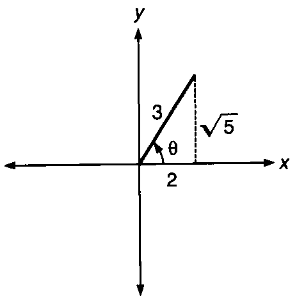
17. $\sin\theta = \dfrac{8\sqrt{89}}{89}$, $\csc\theta = \dfrac{\sqrt{89}}{8}$, $\cos\theta = \dfrac{-5\sqrt{89}}{89}$, $\sec\theta = \dfrac{-\sqrt{89}}{5}$, $\tan\theta = -\dfrac{8}{5}$, $\cot\theta = \dfrac{-5}{8}$



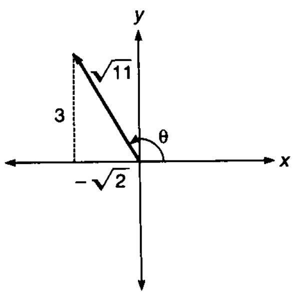
18. $\cos\theta = 0$, $\sec\theta$ undefined, $\sin\theta = -1$, $\csc\theta = -1$, $\tan\theta$ undefined, $\cot\theta = 0$



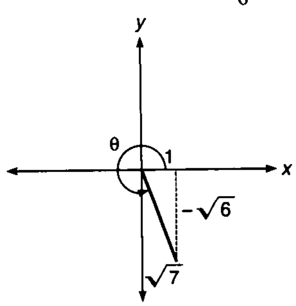
19. $\cos\theta = \frac{2}{3}$, $\sec\theta = \frac{3}{2}$, $\sin\theta = \frac{\sqrt{5}}{3}$, $\csc\theta = \frac{3\sqrt{5}}{5}$, $\tan\theta = \frac{\sqrt{5}}{2}$, $\cot\theta = \frac{2\sqrt{5}}{5}$



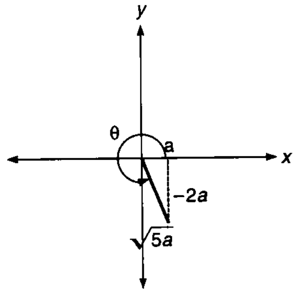
20. $\cos\theta = \frac{-\sqrt{22}}{11}$, $\sec\theta = \frac{-\sqrt{22}}{2}$, $\sin\theta = \frac{3\sqrt{11}}{11}$, $\csc\theta = \frac{\sqrt{11}}{3}$, $\tan\theta = \frac{-3\sqrt{2}}{2}$, $\cot\theta = \frac{-\sqrt{2}}{3}$



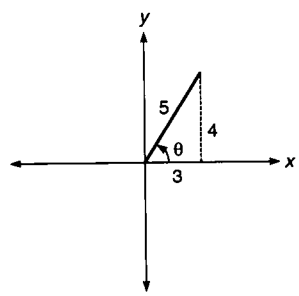
21. $\cos\theta = \frac{\sqrt{7}}{7}$, $\sec\theta = \sqrt{7}$, $\sin\theta = \frac{-\sqrt{42}}{7}$, $\csc\theta = \frac{-\sqrt{42}}{6}$, $\tan\theta = -\sqrt{6}$, $\cot\theta = \frac{-\sqrt{6}}{6}$



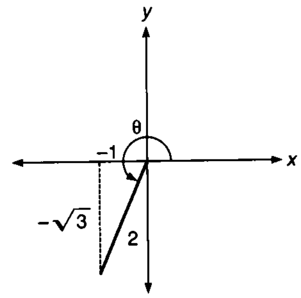
22. $r = \sqrt{a^2 + (-2a)^2} = \sqrt{5a^2} = \sqrt{5}a$
$\sin\theta = \frac{-2a}{\sqrt{5}a} = \frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$,
$\cos\theta = \frac{a}{\sqrt{5}a} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$,
$\tan\theta = \frac{-2a}{a} = -2$, $\csc\theta = -\frac{\sqrt{5}}{2}$, $\sec\theta = \sqrt{5}$, $\cot\theta = -\frac{1}{2}$



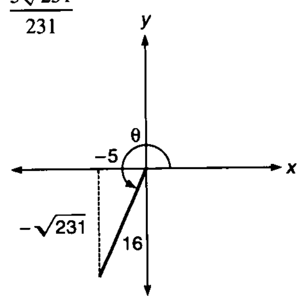
23. III **24.** III **25.** IV **26.** IV
27. II **28.** II **29.** III **30.** IV
31. 46.3° **32.** 36°44′ **33.** 61°48′
34. 22.7° **35.** 68.7° **36.** 74.85°
37. 62.57° **38.** −0.5505 **39.** $-\frac{1}{2}$
40. 2.1842 **41.** 2 **42.** 1.5818
43. 0.6720 **44.** 2 **45.** −0.6862
46. 0.9159 **47.** 161° **48.** 141.0°
49. 212.2° **50.** 294.7°
51. 119.0° **52.** 193.7° **53.** 120°
54. 296.6° **55.** 112.6°
56. $\cos\theta = \frac{3}{5}$, $\sec\theta = \frac{5}{3}$, $\csc\theta = \frac{5}{4}$, $\tan\theta = \frac{4}{3}$, $\cot\theta = \frac{3}{4}$



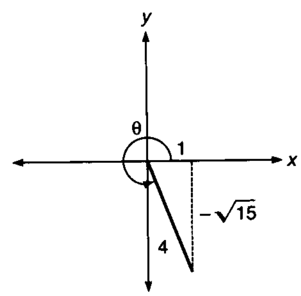
57. $\sec\theta = -2$, $\sin\theta = \frac{-\sqrt{3}}{2}$, $\csc\theta = \frac{-2\sqrt{3}}{3}$, $\tan\theta = \sqrt{3}$, $\cot\theta = \frac{\sqrt{3}}{3}$



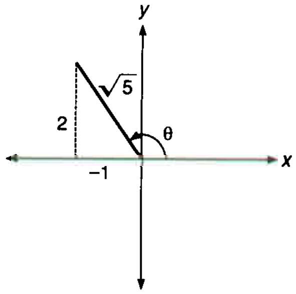
58. $\sec\theta = -\frac{16}{5}$, $\sin\theta = \frac{-\sqrt{231}}{16}$, $\csc\theta = \frac{-16\sqrt{231}}{231}$, $\tan\theta = \frac{\sqrt{231}}{5}$, $\cot\theta = \frac{5\sqrt{231}}{231}$



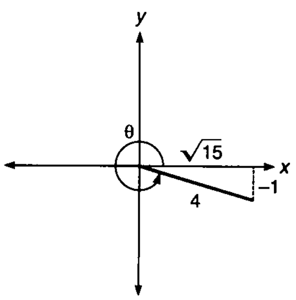
59. $\sec\theta = 4$, $\sin\theta = \frac{-\sqrt{15}}{4}$, $\csc\theta = \frac{-4\sqrt{15}}{15}$, $\tan\theta = -\sqrt{15}$, $\cot\theta = \frac{-\sqrt{15}}{15}$



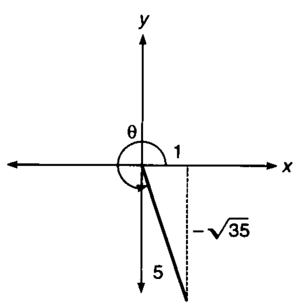
60. $\cot\theta = -\frac{1}{2}$, $\sin\theta = \frac{2\sqrt{5}}{5}$, $\csc\theta = \frac{\sqrt{5}}{2}$, $\cos\theta = -\frac{\sqrt{5}}{5}$, $\sec\theta = -\sqrt{5}$



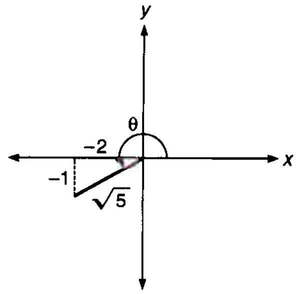
61. $\cos\theta = \frac{\sqrt{15}}{4}$, $\sec\theta = \frac{4\sqrt{15}}{15}$, $\cot\theta = -\sqrt{15}$, $\tan\theta = \frac{-\sqrt{15}}{15}$, $\sin\theta = \frac{-1}{4}$



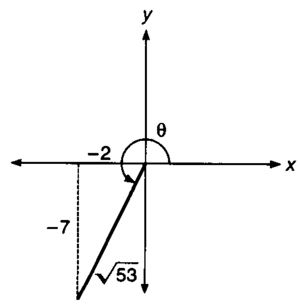
62. $\cos\theta = \frac{1}{6}$, $\sin\theta = \frac{-\sqrt{35}}{6}$, $\csc\theta = \frac{-6\sqrt{35}}{35}$, $\tan\theta = -\sqrt{35}$, $\cot\theta = \frac{-\sqrt{35}}{35}$



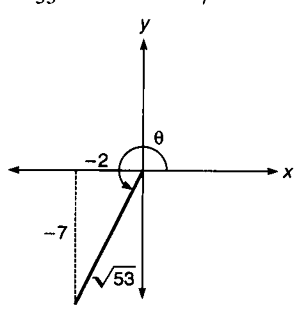
63. $\tan\theta = \frac{1}{2}$, $\sin\theta = \frac{-\sqrt{5}}{5}$, $\cos\theta = \frac{-2\sqrt{5}}{5}$, $\csc\theta = -\sqrt{5}$, $\sec\theta = \frac{-\sqrt{5}}{2}$



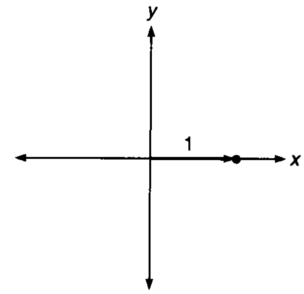
64. $\cot\theta = \frac{2}{7}$, $\sin\theta = \frac{-7\sqrt{53}}{53}$, $\cos\theta = \frac{-2\sqrt{53}}{53}$, $\csc\theta = \frac{-\sqrt{53}}{7}$, $\sec\theta = \frac{-\sqrt{53}}{2}$



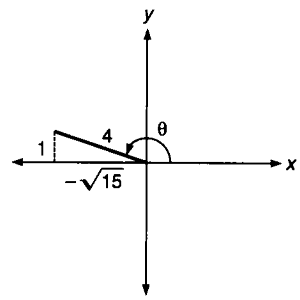
65. $\tan\theta = \frac{7}{2}$, $\sin\theta = \frac{-7\sqrt{53}}{53}$, $\cos\theta = \frac{-2\sqrt{53}}{53}$, $\csc\theta = \frac{-\sqrt{53}}{7}$, $\sec\theta = \frac{-\sqrt{53}}{2}$



66. $\cot\theta$ undefined, $\sin\theta = 0$, $\cos\theta = 1$, $\csc\theta$ undefined, $\sec\theta = 1$



67. $\sin\theta = \frac{1}{4}$, $\cos\theta = \frac{-\sqrt{15}}{4}$, $\tan\theta = \frac{-\sqrt{15}}{15}$, $\sec\theta = \frac{-4\sqrt{15}}{15}$, $\csc\theta = 4$, $\cot\theta = -\sqrt{15}$



68. $\tan\theta = \frac{-z}{\sqrt{1-z^2}}$

69. $\cos\theta = \frac{1}{\sqrt{1+z^2}}$ **70. a.** 31.06 volts **b.** 103.92 volts **c.** 118.46 volts **d.** 10.46 volts **e.** 0 volts **f.** −60 volts **71.** 337.4 m **72.** $x \approx -5.2$ mm, $y \approx 7.3$ mm **73.** $x \approx 7.9$ in., $y \approx 1.4$ in. **74.** $x \approx 5.1$ ft, $y \approx -4.6$ ft **75.** $\frac{2\pi}{3}$, 2.09 **76.** $\frac{-43\pi}{36}$, −3.75 **77.** $\frac{43\pi}{18}$, 7.50 **78.** 630° **79.** 660° **80.** $\frac{-900°}{7}$, −128.57° **81.** $\frac{450°}{\pi}$, 143.24° **82.** $\frac{-756°}{\pi}$, −240.64° **83.** 12.3 inches **84.** 2.3 **85.** $\frac{25\pi}{4}$ mm **86.** 100.8 inches **87.** $\frac{27\pi}{4}$ in.^{2}, 21.21 in.^{2} **88.** $\frac{128\pi}{3}$ mm^2, 134.04 mm^2 **89.** 16.50 mm^2 **90.** $\frac{49\pi}{5}$ in.^{2}, 30.79 in.^{2} **91.** 0.9463

92. -1.3561 **93.** 4.6373 **94.** $\frac{-\sqrt{3}}{2}$

95. -1 **96.** $\frac{-\sqrt{3}}{2}$ **97.** $\frac{\sqrt{3}}{2}$

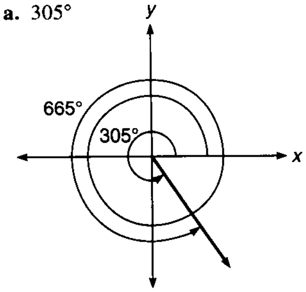
98. $-\sqrt{3}$ **99.** $-\frac{1}{2}$ **100.** $\sqrt{2}$

101. $\frac{11\pi}{6}$ **102.** 2.07 **103.** 0.43

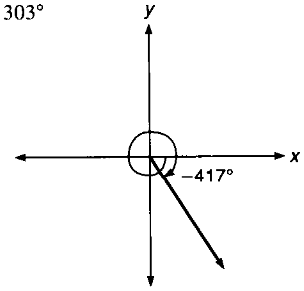
104. 0.5 **105.** $\frac{\pi}{3}$ or 0 **106.** 0.1074

Chapter 2 test

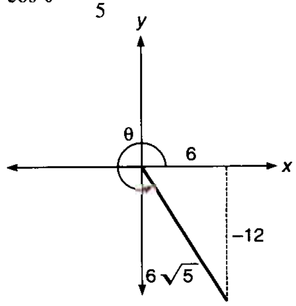
1. a. 305°



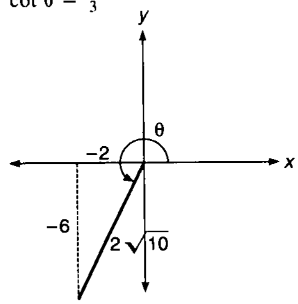
b. 303°



2. $\sin\theta = \frac{-2\sqrt{5}}{5}$, $\sec\theta = \sqrt{5}$, $\tan\theta = -2$, $\csc\theta = \frac{-\sqrt{5}}{2}$, $\cot\theta = -\frac{1}{2}$, $\cos\theta = \frac{\sqrt{5}}{5}$



3. $\sin\theta = \frac{-3\sqrt{10}}{10}$, $\csc\theta = \frac{-\sqrt{10}}{3}$, $\cos\theta = -\frac{\sqrt{10}}{10}$, $\sec\theta = -\sqrt{10}$, $\tan\theta = 3$, $\cot\theta = \frac{1}{3}$

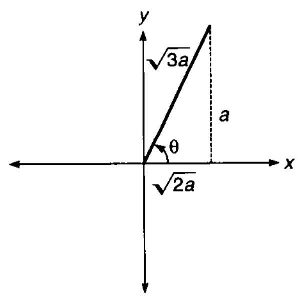


4. $r = \sqrt{(\sqrt{2}a)^2 + a^2} = \sqrt{3a^2} = \sqrt{3}a$

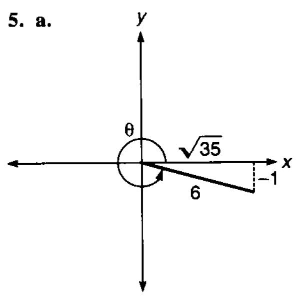
$\sin\theta = \frac{a}{\sqrt{3}a} = \frac{\sqrt{3}}{3}$, $\cos\theta = \frac{\sqrt{2}a}{\sqrt{3}a} = \frac{\sqrt{6}}{3}$,

$\tan\theta = \frac{a}{\sqrt{2}a} = \frac{\sqrt{2}}{2}$, $\csc\theta = \sqrt{3}$,

$\sec\theta = \frac{\sqrt{6}}{2}$, $\cot\theta = \sqrt{2}$

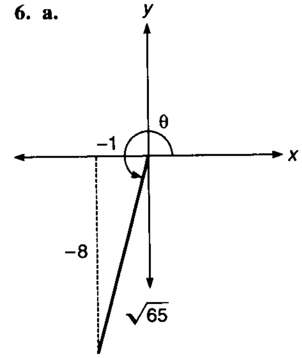


5. a.



b. $\csc\theta = -6$, $\cos\theta = \frac{\sqrt{35}}{6}$, $\sec\theta = \frac{6\sqrt{35}}{35}$, $\tan\theta = \frac{-\sqrt{35}}{35}$, $\cot\theta = -\sqrt{35}$ **c.** 350.4°

6. a.



b. $\cot\theta = \frac{1}{8}$, $\sin\theta = \frac{-8\sqrt{65}}{65}$, $\cos\theta = \frac{-\sqrt{65}}{65}$, $\csc\theta = \frac{-\sqrt{65}}{8}$, $\sec\theta = -\sqrt{65}$ **c.** 262.9° **7.** 66.2°

8. 55.6° **9.** 0.8957 **10.** -0.2642

11. 1.1064 **12.** 197.2° **13.** 119.0°

14. 5.1 amperes

15. $x \approx -3.4$ cm, $y \approx -22.3$ cm

16. a. 108.8° **b.** 655.1 in.

17. $\frac{83\pi}{36}$, 7.24 **18.** 105°

19. 20.5 inches **20.** 14.66 cm

21. 1.6709 **22.** $\frac{-\sqrt{3}}{2}$

23. 134.04 mm^2 **24. a.** yes **b.** yes

c. $\{(-3,2), (5,3), (6,5), (12,10)\}$

25. a. $(-4,33)$ **b.** $(\sqrt{2}, 7 - 3\sqrt{2})$

26. 5.94 **27.** $\frac{4\pi}{3}$ **28.** 1.16

29. 0.38 **30.** 0 or $\frac{\pi}{6}$ **31.** 0.0589

Chapter 3

Exercise 3–1

Answers to odd-numbered problems

1. a. See figure 3–3. **b.** See figure 3–6. **c.** See figure 3–9. **3. a.** $\frac{\pi}{2} + 2k\pi$ **b.** $\frac{3\pi}{2} + 2k\pi$ **c.** $k\pi$ **5.** $k\pi$ **7. a.** $-\frac{1}{2}$ **b.** $\frac{\sqrt{3}}{2}$ **c.** $\frac{-\sqrt{3}}{3}$ **9. a.** $\frac{\sqrt{3}}{2}$ **b.** $\frac{1}{2}$ **c.** $\sqrt{3}$ **11.** odd **13.** even **15.** even **17.** odd **19.** even **21.** even **23.** odd **25.** even

Solutions to trial exercise problems

9. a. $\sin\left(-\frac{5\pi}{3}\right) = -\sin\frac{5\pi}{3}$

Sine is an odd function.

$\sin\frac{5\pi}{3} = -\sin\frac{\pi}{3}$

Sine is negative in quadrant IV.

$= -\frac{\sqrt{3}}{2}$,

so $\sin\left(-\frac{5\pi}{3}\right) = -\sin\frac{5\pi}{3}$

$= -\left(-\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$.

b. $\cos\left(-\frac{5\pi}{3}\right) = \cos\frac{5\pi}{3}$

Cosine is an even function.

$\cos\frac{5\pi}{3} = \cos\frac{\pi}{3}$

Cosine is positive in quadrant IV.

$= \frac{1}{2}$,

so $\cos\left(-\frac{5\pi}{3}\right) = \cos\frac{5\pi}{3} = \frac{1}{2}$.

c. $\tan\left(-\frac{5\pi}{3}\right) = -\tan\frac{5\pi}{3}$

Tangent is an odd function.

$\tan\frac{5\pi}{3} = -\tan\frac{\pi}{3}$

Tangent is negative in quadrant IV.

$= -\sqrt{3}$,

so $\tan\left(-\frac{5\pi}{3}\right) = -\tan\frac{5\pi}{3}$

$= -(-\sqrt{3}) = \sqrt{3}$.

17. $f(x) = 3x - 2x^3$

$f(-x) = 3(-x) - 2(-x)^3$

$= -3x + 2x^3$

$-f(x) = -(3x - 2x^3)$

$= -3x + 2x^3$

so $f(-x) = -f(x)$; therefore, f is an odd function.

Exercise 3–2

Answers to odd-numbered problems

1. See figures 3–11, 3–12, and 3–14.

3. $f(x) = \csc(x) = \frac{1}{\sin x}$

$f(-x) = \frac{1}{\sin(-x)}$

$= \frac{1}{-\sin x}$

$= -\frac{1}{\sin x}$

$= -\csc(x)$

$= -f(x)$

5. $f(x) = \cot x = \frac{1}{\tan x}$

$f(-x) = \cot(-x) = \frac{1}{\tan(-x)}$

$= \frac{1}{-\tan(x)}$

$= -\frac{1}{\tan x}$

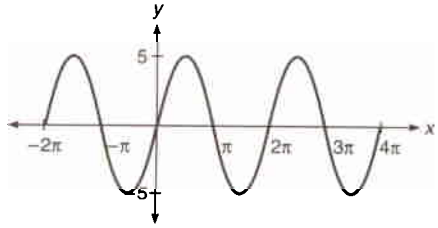
$= -\cot x$

$= -f(x)$

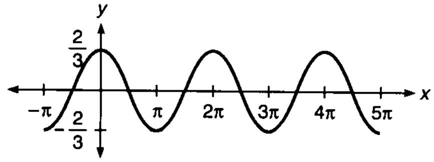
Exercise 3–3

Answers to odd-numbered problems

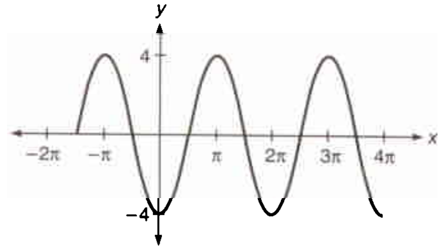
1.



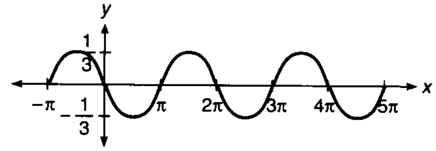
3.



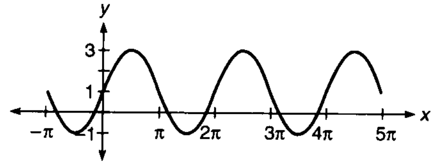
5.



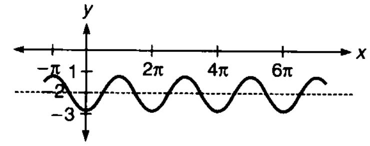
7.



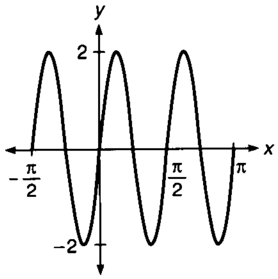
9.



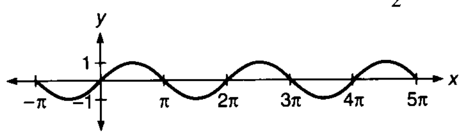
11.



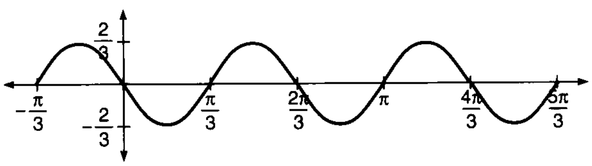
13. amplitude = 2, period = $\frac{\pi}{2}$, phase shift = 0



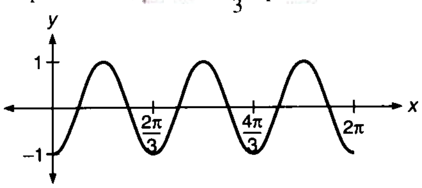
15. amplitude = 1, period = 2π, phase shift = $\frac{\pi}{2}$



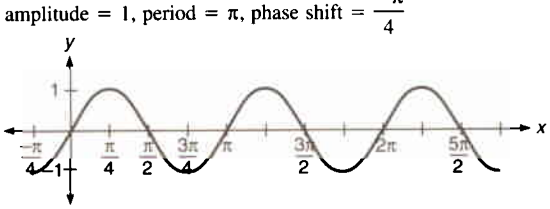
17. amplitude = $\frac{2}{3}$, period = $\frac{2\pi}{3}$, phase shift = $\frac{-\pi}{3}$



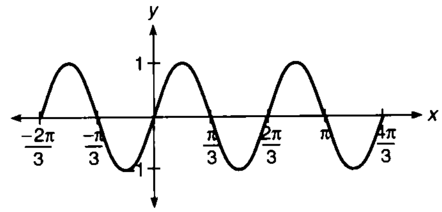
19. amplitude = 1, period = $\frac{2\pi}{3}$, phase shift = 0



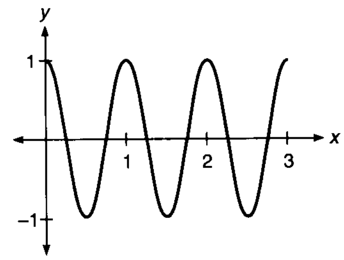
21. amplitude = 1, period = π, phase shift = $\frac{-\pi}{4}$



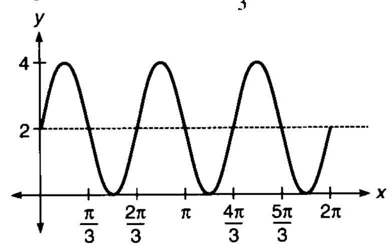
23. amplitude = 1, period = $\frac{2\pi}{3}$, phase shift = $\frac{-2\pi}{3}$



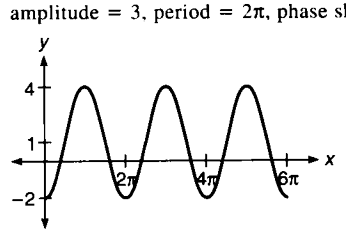
25. amplitude = 1, period = 1, phase shift = 0



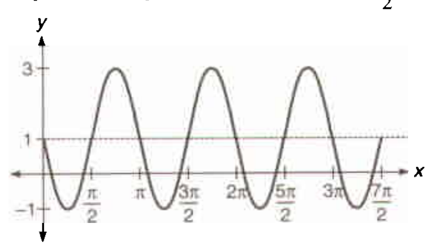
27. amplitude = 2, period = $\frac{2\pi}{3}$, phase shift = 0



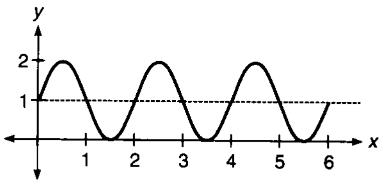
29. amplitude = 3, period = 2π, phase shift = 0



31. amplitude = 2, period = π, phase shift = $\frac{\pi}{2}$



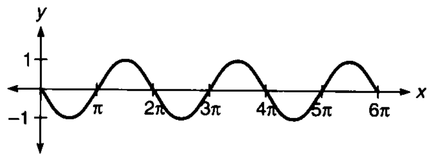
33. amplitude = 1, period = 2, phase shift = 0



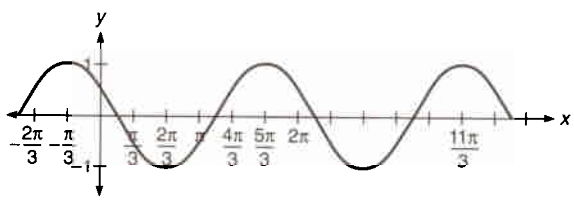
35. $y = -\sin 2x$ **37.** $y = -\cos 3x$ **39.** $y = -\sin(x + 3)$

41. $y = -\sin x - 3$ **43.** $y = -3 \cos\left(2x - \dfrac{\pi}{2}\right)$

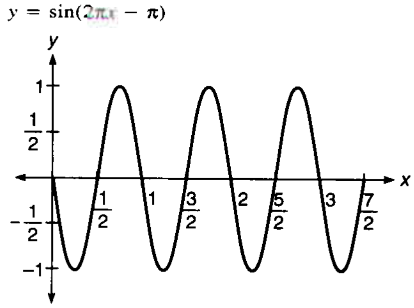
45. $y = -\sin x$



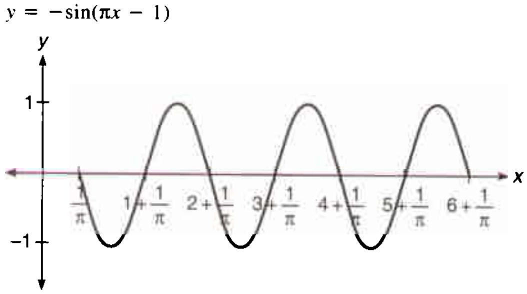
47. $y = \cos\left(x + \dfrac{\pi}{3}\right)$



49. $y = \sin(2\pi x - \pi)$



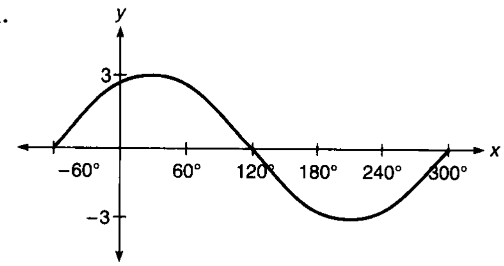
51. $y = -\sin(\pi x - 1)$



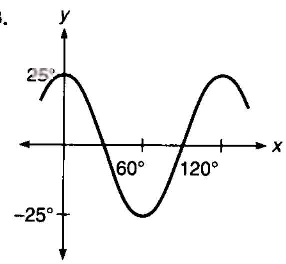
53. $A = 3, B = 4, C = 0, D = 0$ **55.** $A = 2, B = \dfrac{2\pi}{5}$, $C = \dfrac{-2\pi}{5}, D = 0$ **57.** $A = 3, B = 4, C = \dfrac{-\pi}{2}, D = 0$

59. $A = 2, B = \dfrac{2\pi}{5}, C = \dfrac{-9\pi}{10}, D = 0$

61.



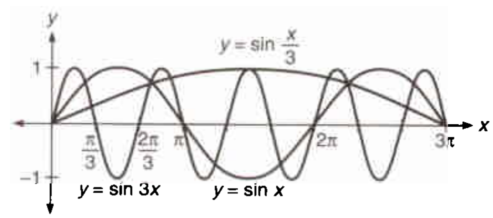
63.



65. $y = 60 \sin\left(\dfrac{20x}{3} - 600\right)$

67. $y = 0.5 \sin 43x + 23.5$

69.

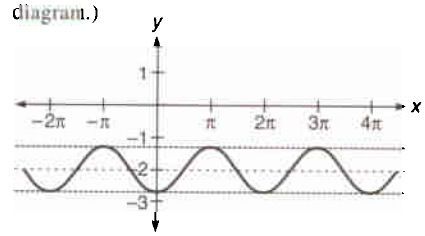


71. Both graphs are the same as figure 3–3, for the sine function.

Solutions to trial exercise problems

11. $y = -\dfrac{3}{4} \cos x - 2$

Period is 2π because the coefficient of the argument is 1. The amplitude is $\frac{3}{4}$, with a reflection about the horizontal axis. There is a vertical shift of 2 units downward. (See the diagram.)



21. $y = -\cos\left(2x + \frac{\pi}{2}\right)$

$0 \le 2x + \frac{\pi}{2} \le 2\pi$

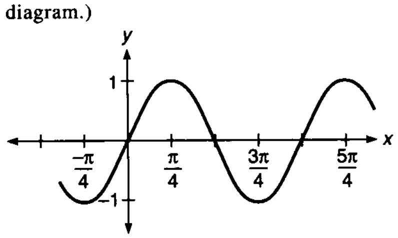
Subtract $\frac{\pi}{2}$.

$-\frac{\pi}{2} \le 2x \le \frac{3\pi}{2}$

Multiply by $\frac{1}{2}$.

$-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$

Thus we get one complete cycle of the cosine function, reflected about the horizontal axis (because of the negative coefficient), starting at $-\frac{\pi}{4}$ and ending at $\frac{3\pi}{4}$. (See the diagram.)



The period is $\frac{3\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$.

Phase shift is $-\frac{\pi}{4}$, and amplitude is 1. We mark the x-axis in increments of one half the period, or $\frac{\pi}{2}$, starting at $-\frac{\pi}{4}$.

43. $y = -3\cos\left(-2x + \frac{\pi}{2}\right)$

Since cosine is an even function, we merely change the sign of the argument. $y = -3\cos\left(2x - \frac{\pi}{2}\right)$.

51. $y = \sin(-\pi x + 1)$

Sine is an odd function, so we change both the sign of the coefficient and the sign of the argument. $y = -\sin(\pi x - 1)$.

$0 \le \pi x - 1 \le 2\pi$

Add 1.

$1 \le \pi x \le 2\pi + 1$

Divide by π.

$\frac{1}{\pi} \le x \le \frac{2\pi + 1}{\pi}$

Phase shift is $\frac{1}{\pi}$, period is $\frac{2\pi + 1}{\pi} - \frac{1}{\pi} = \frac{2\pi}{\pi} + \frac{1}{\pi} - \frac{1}{\pi}$ $= 2$. Amplitude is 1.

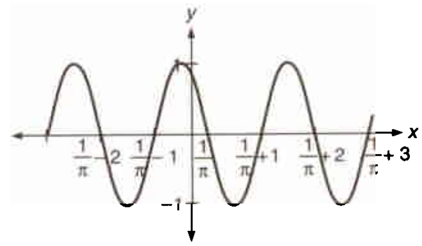
We mark the x-axis in increments of one half the period (1), starting at $\frac{1}{\pi}$. (We also compute decimal approximations as a convenience in plotting.) The calculations are as follows:

$\frac{1}{\pi} + 1 \approx 1.3$; $\left(\frac{1}{\pi} + 1\right) + 1 = \frac{1}{\pi} + 2 \approx 2.3$;

$\left(\frac{1}{\pi} + 2\right) + 1 = \frac{1}{\pi} + 3 \approx 3.3$;

$\frac{1}{\pi} + 4 \approx 4.3$; $\frac{1}{\pi} + 5 \approx 5.3$; $\left(\frac{1}{\pi} + 1\right) - 1 = \frac{1}{\pi} \approx 0.3$;

$\frac{1}{\pi} - 1 \approx -0.7$. (See the diagram.)



56. Amplitude = 2. We have one cycle between 0 and 3π.

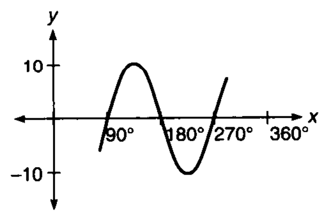
$0 \le x \le 3\pi$

Multiply by $\frac{2}{3}$.

$0 \le \frac{2x}{3} \le 2\pi$

Vertical shift = 3, so the equation is $y = 2\sin\left(\frac{2x}{3}\right) + 3$.

64. $y = 10\sin(2x - 180°)$ (See the diagram.)



$0° \le 2x - 180° \le 360°$

Add 180°.

$180° \le 2x \le 540°$

Divide by 2.

$90° \le x \le 270°$

67. Amplitude is 0.5; 0° phase shift; period $\left(\frac{360}{43}\right)^{\circ}$; vertical translation 23.5.

$0° \le x \le \frac{360°}{43}$

$0° \le 43x \le 360°$

Thus, the argument is $43x$, and we have for $y = A\sin(Bx + C) + D$,

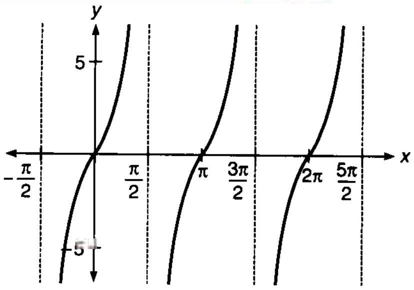
$A = 0.5$, $B = 43$, $C = 0$, $D = 23.5$, for

$y = 0.5\sin 43x + 23.5$

Exercise 3–4

Answers to odd-numbered problems

1.



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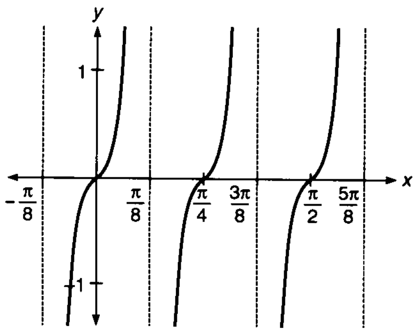

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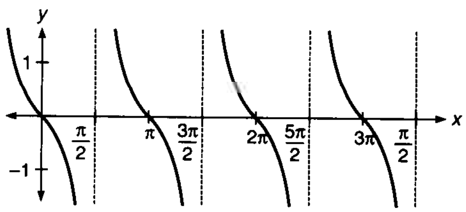

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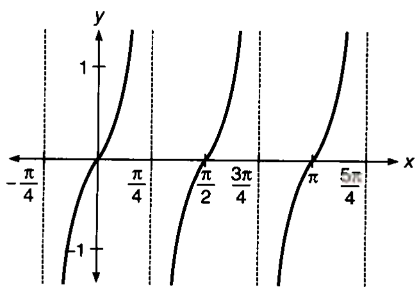

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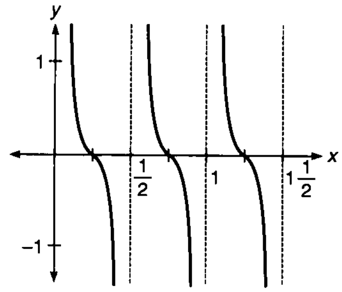
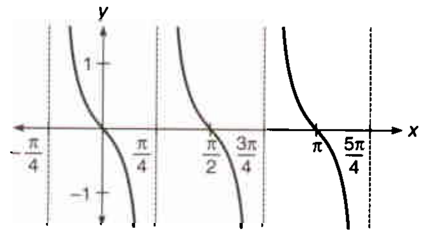
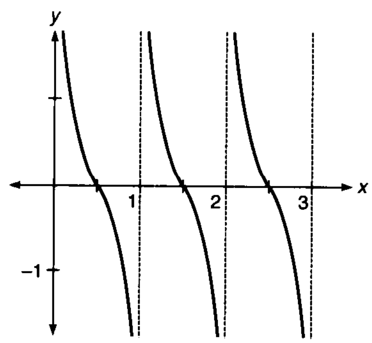


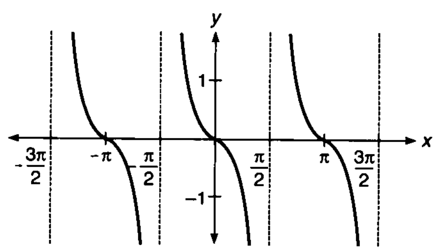
11. $y = -\tan 2x$



13. $y = \cot \pi x$



15. $y = -\tan(x + \pi)$



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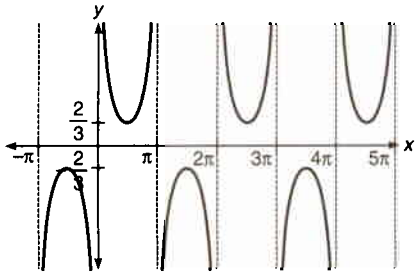

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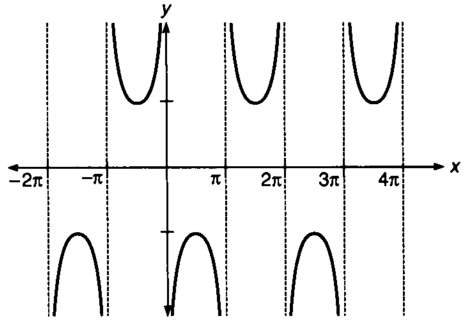

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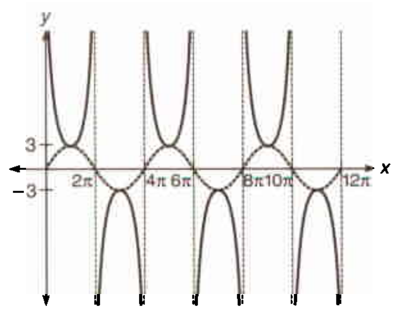

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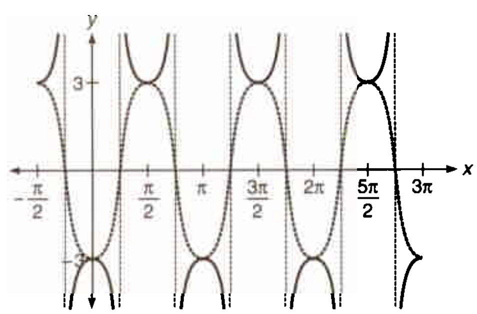

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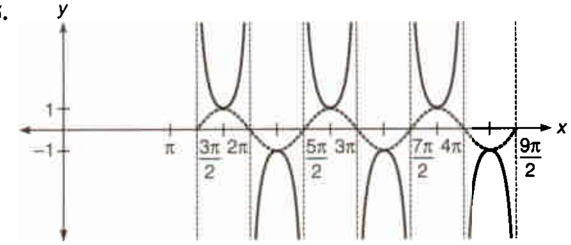

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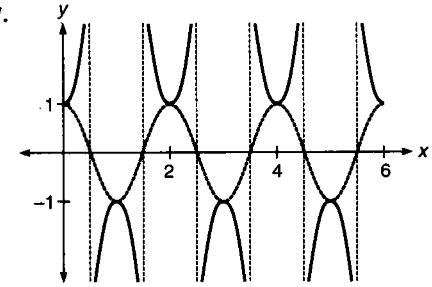

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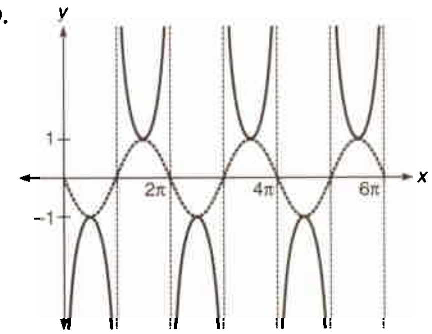

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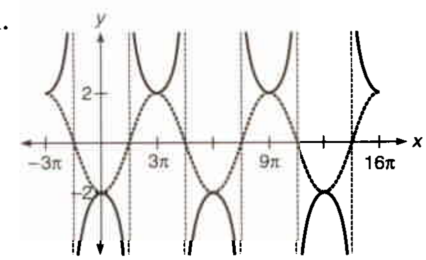


Solutions to trial exercise problems

7. $y = -\cot\left(2x + \frac{\pi}{2}\right)$

There is a reversal about the horizontal axis, because of the negative coefficient. To find the beginning and end points for one cycle,

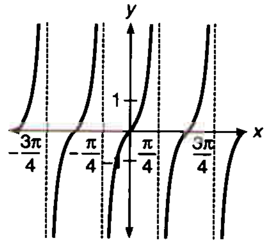
$0 < 2x + \frac{\pi}{2} < \pi$

Subtract $\frac{\pi}{2}$.

$-\frac{\pi}{2} < 2x < \frac{\pi}{2}$

Multiply by $\frac{1}{2}$.

$-\frac{\pi}{4} < x < \frac{\pi}{4}$



Thus, one complete cycle starts at $\frac{-\pi}{4}$ and terminates at $\frac{\pi}{4}$, reversed about the horizontal axis. (See the diagram.)

15. $y = \tan(-x - \pi)$

The tangent function is odd, so we change both the sign of the argument and the coefficient of the function.

$y = -\tan(x + \pi)$

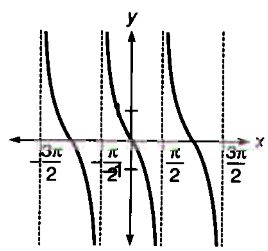
We find the beginning and end points.

$-\frac{\pi}{2} < x + \pi < \frac{\pi}{2}$

Subtract π.

$-1\frac{1}{2}\pi < x < -\frac{1}{2}\pi$.

See the diagram.



24. $y = \frac{2}{3}\sec(3x + \pi)$

We first graph $y = \frac{2}{3}\cos(3x + \pi)$.

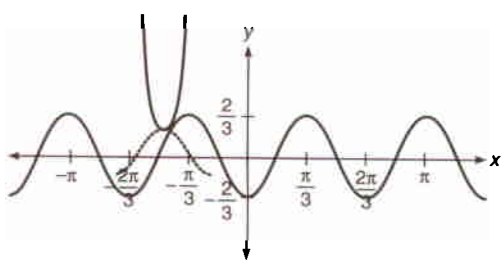
$0 \le 3x + \pi \le 2\pi$

Subtract π.

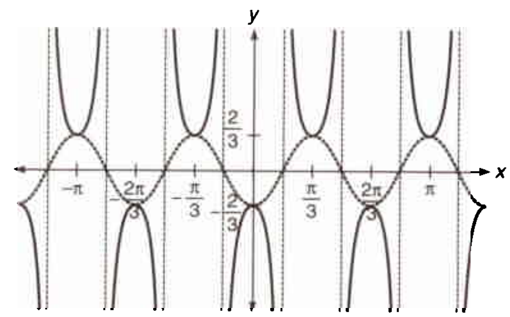
$-\pi \le 3x \le \pi$

$-\frac{\pi}{3} \le x \le \frac{\pi}{3}$

See the diagram.



Next we construct the graph of the corresponding secant function. This graph touches the cosine graph at its high and low values and has vertical asymptotes wherever the cosine graph is 0 (crosses the x-axis). (See the diagram.)

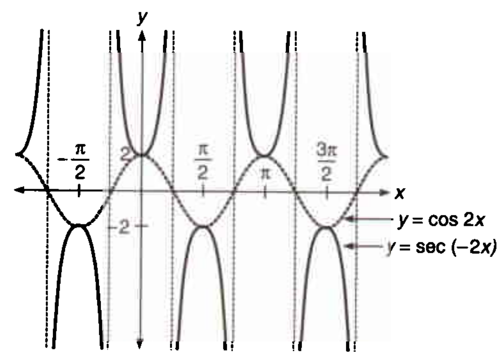


28. $y = \sec(-2x)$

The secant function is even (as is the cosine function), so we simply change the sign of the argument.

$y = \sec 2x$

We graph $y = \cos 2x$ first, then use this to construct the secant graph. (See the diagram.)



$0 \le 2x \le 2\pi$

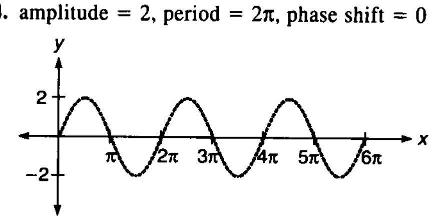
$0 \le x \le \pi$

Chapter 3 review

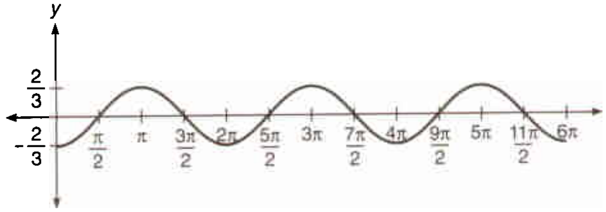
1. See figure 3-3 for the graph; R; $-1 \le y \le 1$; 2π **2.** all multiples of 2π: $2k\pi$, k an integer **3.** $\frac{\sqrt{3}}{2}$ **4.** $-\sqrt{3}$ **5.** $-\frac{1}{2}$ **6.** $\sqrt{2}$ **7.** odd **8.** even **9.** odd **10.** See figure 3-11. **11.** domain: $x \neq k\pi$, k an integer; range: R **12.** $f(-x) = \frac{-x}{\sec(-x)} = \frac{-x}{\sec x} = -f(x)$

13. $f(-x) = \sec(-x) \cdot [\sin(-x)]^2 + (-x)^4 = \sec x \cdot [-\sin x]^2 + x^4 = \sec x \cdot \sin^2 x + x^4 = f(x)$

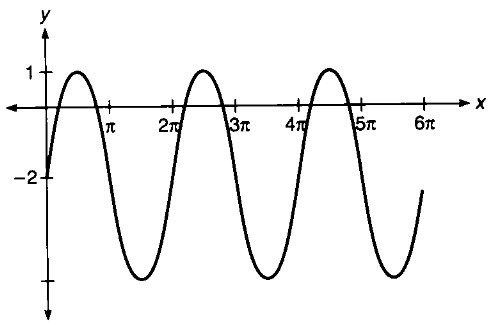
14. amplitude = 2, period = 2π, phase shift = 0



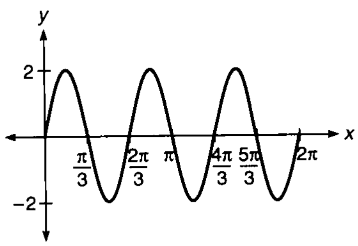
15. amplitude $= \frac{2}{3}$, period $= 2\pi$, phase shift $= 0$



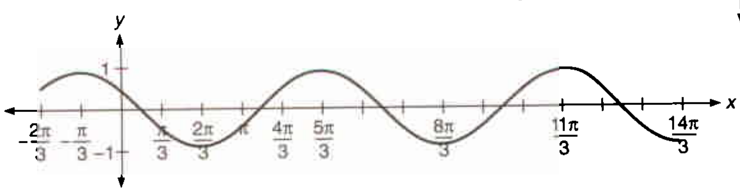
16. amplitude $= 3$, period $= 2\pi$, phase shift $= 0$



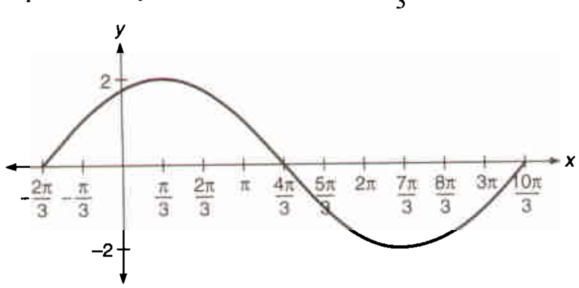
17. amplitude $= 2$, period $= \frac{2\pi}{3}$, phase shift $= 0$



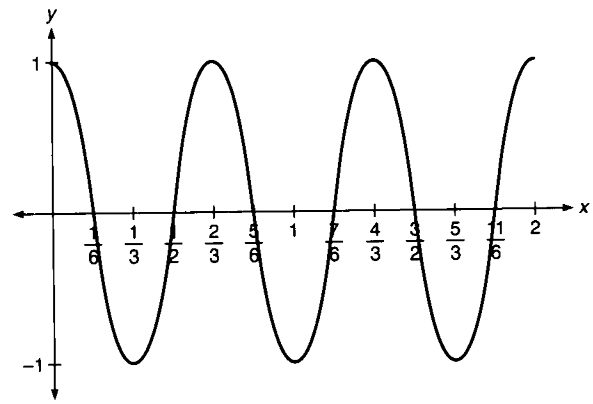
18. amplitude $= 1$, period $= 2\pi$, phase shift $= \frac{-\pi}{3}$



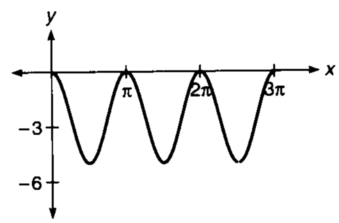
19. amplitude $= 2$, period $= 4\pi$, phase shift $= \frac{-2\pi}{3}$



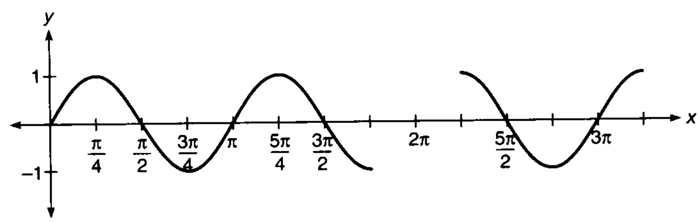
20. amplitude $= 1$, period $= \frac{2}{3}$, phase shift $= 0$



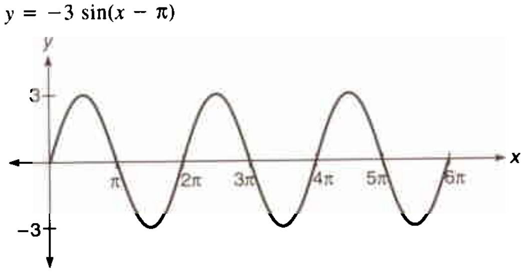
21. amplitude $= 3$, period $= \pi$, phase shift $= 0$



22. $y = \cos\left(2x - \frac{\pi}{2}\right)$



23. $y = -3\sin(x - \pi)$



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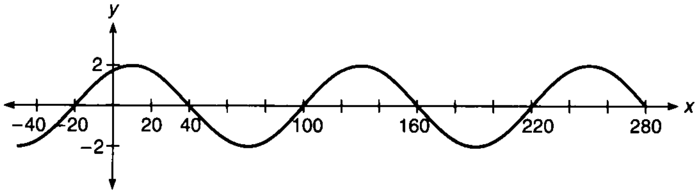
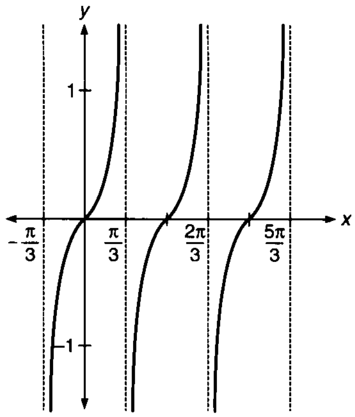


25. amplitude = 2, so $A = 2$; $Bx + C = 2x - \frac{\pi}{3}$, so $B = 2$, $C = -\frac{\pi}{3}$, and $D = -1$; $y = 2\cos\left(2x - \frac{\pi}{3}\right) - 1$

26. $y = \sin\left(\frac{2\pi}{23}x + \frac{20\pi}{23}\right)$

27.



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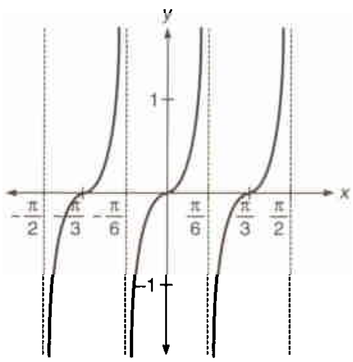

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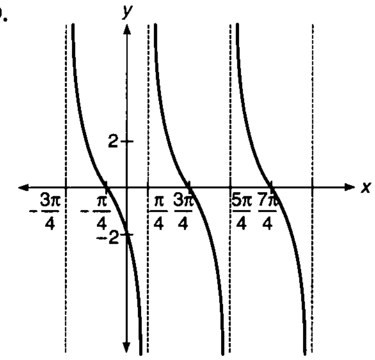

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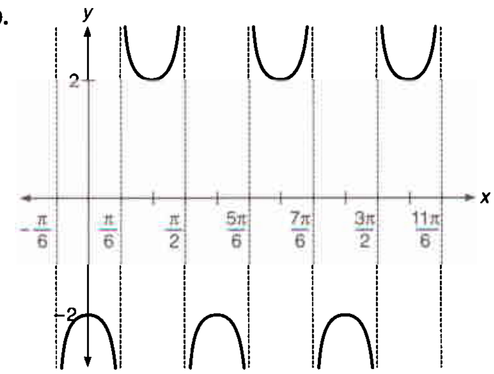

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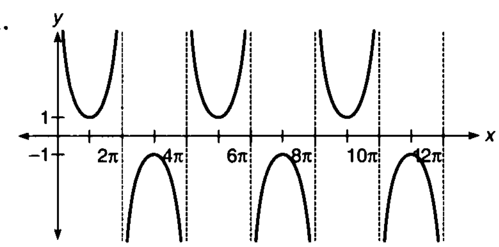

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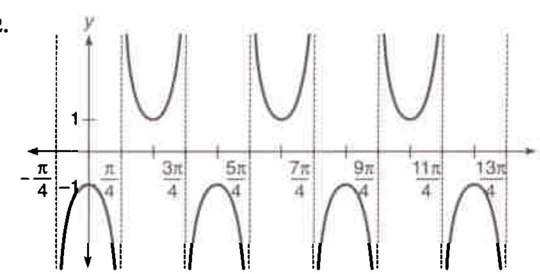

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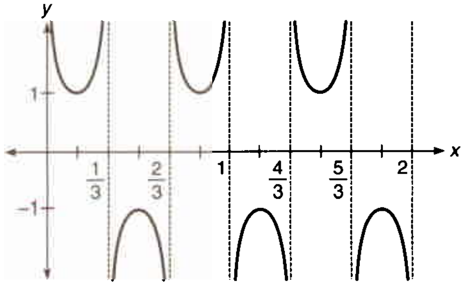
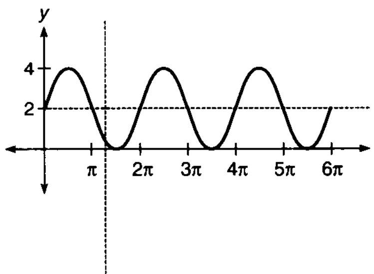


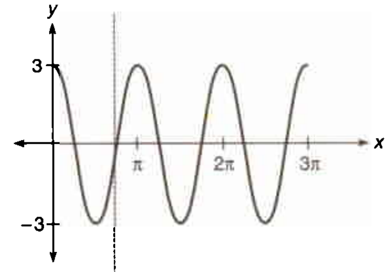
Chapter 3 test

1. $\frac{3\pi}{2} + 2k\pi$, k an integer **2.** $\sqrt{3}$ **3.** odd

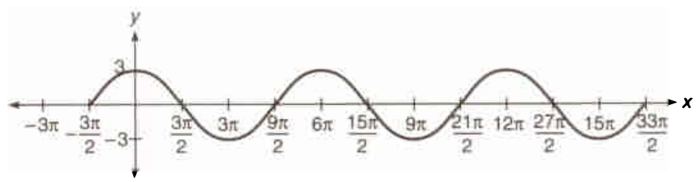
4. amplitude = 2, period = 2π, phase shift = 0



5. amplitude = 3, period = π, phase shift = 0



6. amplitude = 3, period = 6π, phase shift = $\frac{-3\pi}{2}$



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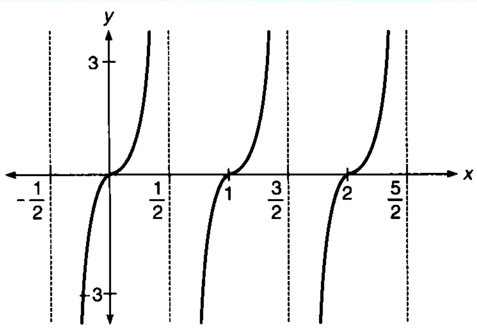

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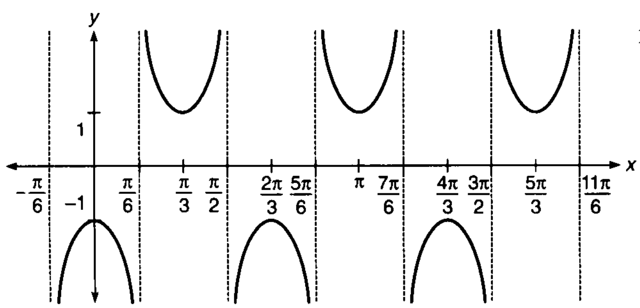

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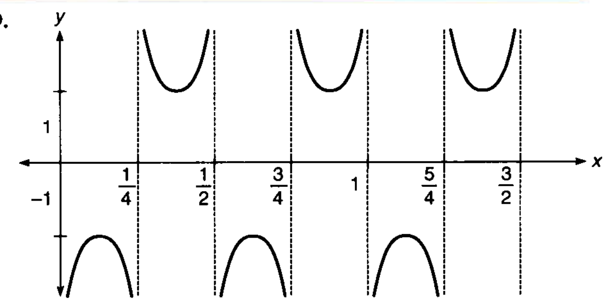
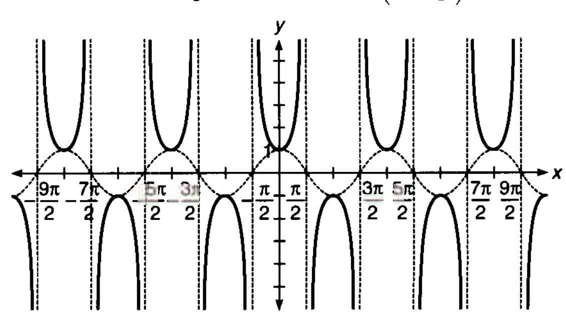


10. $A = 2$, $B = 1$, $C = \frac{\pi}{3}$, $D = 0$; $y = 2\sin\left(x + \frac{\pi}{3}\right)$.

11.



12. $f(-x) = \sec(-x) \cdot \sin(-x) + (-x)^3 = \sec x \cdot (-\sin x) - x^3 = -(\sec x \cdot \sin x + x^3) = -f(x)$

13. $y = \sin\left(\frac{\pi}{14}x - \frac{5\pi}{14}\right)$

14. $y = 25\sin\left(\frac{12}{5}x + 48°\right) + 10$

Chapter 4

Exercise 4–1

Answers to odd-numbered problems

1. Let $y = f(x) = 2x - 7$. Show $g(y) = x$:
$g(y) = \frac{1}{2}y + 3\frac{1}{2}$
$= \frac{1}{2}(2x - 7) + \frac{7}{2}$
$= x$
Let $y = g(x) = \frac{1}{2}x + \frac{7}{2}$. Show $f(y) = x$:
$f(y) = 2y - 7 = 2(\frac{1}{2}x + \frac{7}{2}) - 7 = x$

3. Let $y = f(x) = \frac{1}{3}x + \frac{8}{3}$. Show $g(y) = x$:
$g(y) = 3y - 8 = 3(\frac{1}{3}x + \frac{8}{3}) - 8 = x$
Let $y = g(x) = 3x - 8$. Show $f(y) = x$:
$f(y) = \frac{1}{3}y + \frac{8}{3} = \frac{1}{3}(3x - 8) + \frac{8}{3} = x$

5. Let $y = f(x) = 2x - 5$. Show $g(y) = x$:
$g(y) = \frac{1}{2}(y + 5) = \frac{1}{2}[(2x - 5) + 5] = x$
Let $y = g(x) = \frac{1}{2}(x + 5)$. Show $f(y) = x$:
$f(y) = 2y - 5 = 2[\frac{1}{2}(x + 5)] - 5 = x$

7. Let $y = f(x) = \frac{2}{x - 3}$. Show $g(y) = x$:

$$g(y) = \frac{2}{y} + 3 = \frac{2}{\frac{2}{x-3}} + 3 = (x - 3) + 3 = x$$

Let $y = g(x) = \frac{2}{x} + 3$. Show $f(y) = x$:

$$f(y) = \frac{2}{y - 3} = \frac{2}{\left(\frac{2}{x} + 3\right) - 3} = \frac{2}{\frac{2}{x}} = x$$

9. Let $y = f(x) = 7 - \frac{3}{x}$. Show $g(y) = x$:

$$g(y) = \frac{3}{7 - y} = \frac{3}{7 - \left(7 - \frac{3}{x}\right)} = \frac{3}{\frac{3}{x}} = x$$

Let $y = g(x) = \frac{3}{7 - x}$. Show $f(y) = x$:

$$f(y) = 7 - \frac{3}{y} = 7 - \frac{3}{\frac{3}{7-x}} = 7 - (7 - x) = x$$

11. Let $y = f(x) = \frac{x}{x - 1}$. Show $g(y) = x$:

$$g(y) = \frac{y}{y - 1} = \frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1}$$

$$= \frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} \cdot \frac{x - 1}{x - 1}$$

$$= \frac{x}{x - (x - 1)} = x$$

Let $y = g(x) = \frac{x}{x - 1}$. Show $f(y) = x$:

$$f(y) = \frac{y}{y - 1} = \frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1}$$

$$= \frac{\frac{x}{x-1}}{\frac{x}{x-1} - 1} \cdot \frac{x - 1}{x - 1}$$

$$= \frac{x}{x - (x - 1)} = x$$

13. Let $y = f(x) = x^2 - 9$. Show $g(y) = x$:
$g(y) = \sqrt{y + 9} = \sqrt{(x^2 - 9) + 9}$
$= \sqrt{x^2} = x$ if $x \geq 0$.
Let $y = g(x) = \sqrt{x + 9}$. Show $f(y) = x$:
$f(y) = y^2 - 9 = (\sqrt{x + 9})^2 - 9$
$= (x + 9) - 9 = x$

15. Let $y = f(x) = x^3$ Show $g(y) = x$:
$g(y) = \sqrt[3]{y} = \sqrt[3]{x^3} = x$
Let $y = g(x) = \sqrt[3]{x}$. Show $f(y) = x$:
$f(y) = y^3 = (\sqrt[3]{x})^3 = x$

17. Let $y = f(x) = x^2 - 2x + 3$. Show $g(y) = x$:
$g(y) = \sqrt{y - 2} + 1$
$= \sqrt{(x^2 - 2x + 3) - 2} + 1$
$= \sqrt{x^2 - 2x + 1} + 1$
$= \sqrt{(x - 1)^2} + 1 = x - 1 + 1 = x$
Let $y = g(x) = \sqrt{x - 2} + 1$. Show $f(y) = x$:
$f(y) = y^2 - 2y + 3$
$= (\sqrt{x - 2} + 1)^2 - 2(\sqrt{x - 2} + 1) + 3$
$= [(x - 2) + 2\sqrt{x - 2} + 1] - 2\sqrt{x - 1} - 2 + 3$
$= x$

19. Let $y = f(x) = \dfrac{2x}{x-3}$. Show $g(y) = x$:

$$g(y) = \frac{3y}{y-2} = \frac{3 \cdot \dfrac{2x}{x-3}}{\dfrac{2x}{x-3} - 2}$$

$$= \frac{3 \cdot \dfrac{2x}{x-3}}{\dfrac{2x}{x-3} - 2} \cdot \frac{x-3}{x-3}$$

$$= \frac{3(2x)}{2x - 2(x-3)} = \frac{6x}{6} = x$$

Let $y = g(x) = \dfrac{3x}{x-2}$. Show $f(y) = x$:

$$f(y) = \frac{2y}{y-3} = \frac{2 \cdot \dfrac{3x}{x-2}}{\dfrac{3x}{x-2} - 3}$$

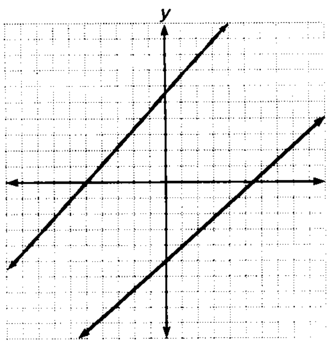
$$= \frac{2 \cdot \dfrac{3x}{x-2}}{\dfrac{3x}{x-2} - 3} \cdot \frac{x-2}{x-2}$$

$$= \frac{6x}{3x - 3(x-2)} = \frac{6x}{6} = x$$

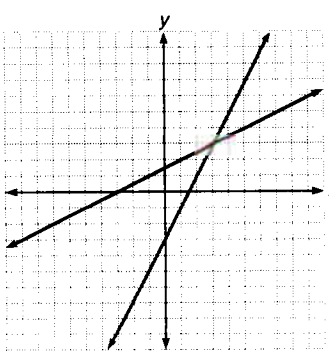
21. function, not one to one

23. not a function

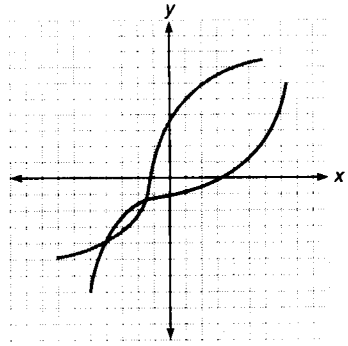
25.



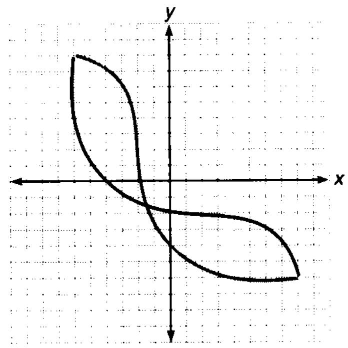
27.



29.



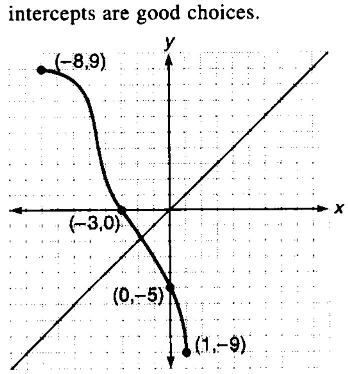
31.



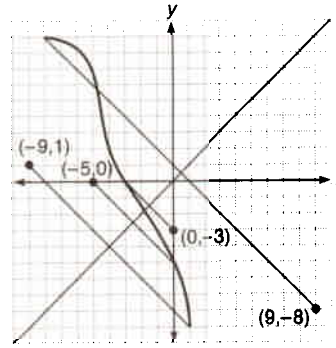
33. $A^{-1}(x) = \dfrac{x}{4} - 4$ **35.** $R^{-1}(x) = \dfrac{20x}{20 - x}$

Solutions to trial exercise problems

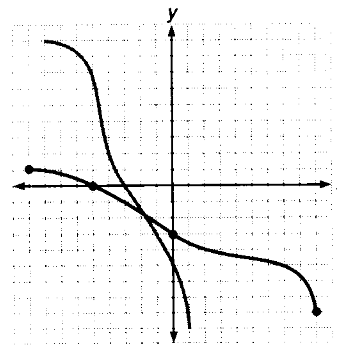
31. We mark four points on the graph. Any points will do, but the end points and intercepts are good choices.



We move these points across the line $y = x$. This is shown in the second figure.



We connect the points and obtain the graph of the inverse function.



34. $d(t) = 16t^2$

$y = 16t^2$

$t = 16y^2$

$y^2 = \dfrac{t}{16}$

$y = \pm\sqrt{\dfrac{t}{16}}$

$y = \pm\dfrac{\sqrt{t}}{4}$

$y = \dfrac{\sqrt{t}}{4}$ (choose + among $\pm\dfrac{\sqrt{t}}{4}$; to see why, consider that the point (1,16) is in the function d, so the point (16,1) must be in d^{-1}).

$d^{-1}(t) = \dfrac{\sqrt{t}}{4}$

Exercise 4–2

Answers to odd-numbered problems

1. See figure 4–6. **3.** $-\frac{\pi}{6}$, $-30°$
5. 0, 0° **7.** $-\frac{\pi}{3}$, $-60°$
9. 1.08 rad, 61.9° **11.** 1.20 rad, 68.8°
13. -1.50 rad, $-86.0°$ **15.** -0.26 rad, $-14.9°$ **17.** $\frac{5\sqrt{39}}{39}$ **19.** $\frac{3\sqrt{5}}{5}$
21. $\frac{\sqrt{66}}{3}$ **23.** $\frac{\sqrt{91}}{10}$ **25.** $\sqrt{1 - z^2}$
27. $\frac{1 + z}{\sqrt{-2z - z^2}}$ **29.** $\frac{\sqrt{1 - 2z}}{1 - 2z}$ **31.** $\frac{\pi}{6}$
33. $-\frac{\pi}{6}$ **35.** $\frac{\pi}{2}$ **37.** Let f be a periodic function; then $f(x) = y = f(x + kp) = y$ gives ordered pairs (x,y), $(x + kp,y)$; second element repeats, so not one to one.

Solutions to trial exercise problems

7. $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$

We want the angle in quadrant I or quadrant IV whose sine value is $-\frac{\sqrt{3}}{2}$.

Since the argument is negative, we know that we want an angle in quadrant IV, where the sine function is negative. Since we have memorized the fact that $\sin 60° = \frac{\sqrt{3}}{2}$, the reference angle is 60°. Therefore, the angle is $-60°$, or $-\frac{\pi}{3}$ (radians).

(Remember that we use negative values for the inverse sine function in quadrant IV.)

13. $\sin^{-1}(-0.9976)$

Calculator:

The keystrokes are the same for radian and degree mode.

.9976 $\boxed{+/-}$ $\boxed{\text{INV}}$ $\boxed{\text{SIN}}$

$\boxed{\text{TI-81}}$ $\boxed{\text{SIN}^{-1}}$ $\boxed{(-)}$ .9976

$\boxed{\text{ENTER}}$

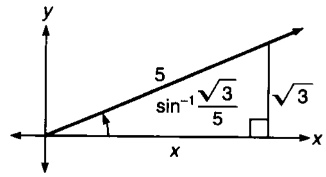
($= -1.501500431 = -1.50$ to two decimal places)

($= -86.02963761° = -86.0°$ to one decimal place)

21. $\cot\left(\sin^{-1}\frac{\sqrt{3}}{5}\right)$

We want the cotangent of a first quadrant angle whose sine is $\frac{\sqrt{3}}{5}$.

Such an angle is shown in the diagram. The Pythagorean theorem shows that $x = \sqrt{22}$, so the cotangent of this angle is $\frac{\text{adj}}{\text{opp}} = \frac{\sqrt{22}}{\sqrt{3}} = \frac{\sqrt{66}}{3}$. (Multiply numerator and denominator by $\sqrt{3}$.)



27. $\tan[\sin^{-1}(1 + z)]$, $1 + z < 0$

The argument of the inverse sine function is negative, so this represents an angle in quadrant IV. The diagram shows a reference triangle in quadrant IV, where the sine of the angle in standard position is $1 + z$. Now we use the Pythagorean theorem to find x.

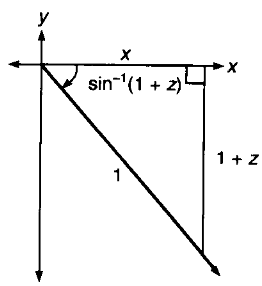
$1^2 = x^2 + (1 + z)^2$
$1 = x^2 + 1 + 2z + z^2$
$-2z - z^2 = x^2$
$\sqrt{-2z - z^2} = x$

Now, the tangent of the angle is

$\frac{\text{opp}}{\text{adj}} = \frac{1 + z}{\sqrt{-2z - z^2}}$

Thus, $\tan[\sin^{-1}(1 + z)]$

$= \frac{1 + z}{\sqrt{-2z - z^2}}$.



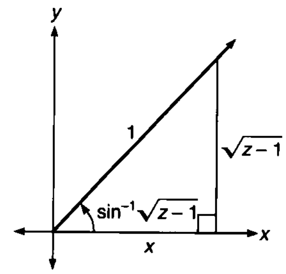
30. $\cot(\sin^{-1}\sqrt{z - 1})$

If $\sqrt{z - 1} = 0$, we know $\sin^{-1}0 = 0$, and cot 0 is undefined. Since $\sqrt{z - 1} < 0$ is impossible, we assume $\sqrt{z - 1} > 0$. In this case, we want a first-quadrant reference triangle. This is shown in the diagram. We find x.

$x^2 + (\sqrt{z - 1})^2 = 1^2$
$x^2 + z - 1 = 1$
$x^2 = 2 - z$
$x = \sqrt{2 - z}$

The cotangent of this angle is

$\frac{\text{adj}}{\text{opp}} = \frac{\sqrt{2 - z}}{\sqrt{z - 1}}$.



34. $\sin^{-1}\left(\sin\frac{11\pi}{6}\right)$

$\sin^{-1}\left(\sin\frac{11\pi}{6}\right) = \sin^{-1}\left(-\frac{1}{2}\right)$

$= -\frac{\pi}{6}$ or $-30°$

Exercise 4–3

Answers to odd-numbered problems

1. a. See figure 4–6. **b.** See figure 4–8.
c. See figure 4–10. **3.** $\frac{2\pi}{3}$ rad, 120°
5. $\frac{\pi}{2}$ rad, 90° **7.** $\frac{\pi}{3}$ rad, 60°
9. $-\frac{\pi}{3}$ rad, $-60°$ **11.** $-\frac{\pi}{6}$ rad, $-30°$
13. 1.08 rad, 61.9° **15.** 0.60 rad, 34.4°
17. 0.75 rad, 43.0° **19.** 0.92 rad, 52.8°
21. -1.50 rad, $-86.0°$ **23.** -0.25 rad, $-14.3°$ **25.** 3.00 rad, 172° **27.** $\frac{5\sqrt{34}}{34}$
29. $\frac{\sqrt{39}}{8}$ **31.** $-\frac{\sqrt{5}}{2}$ **33.** $\frac{\sqrt{30}}{6}$

35. $\frac{10\sqrt{109}}{109}$ **37.** $\sqrt{1 - z^2}$

39. $\frac{\sqrt{1 - z^2}}{z}$ **41.** $\frac{1}{\sqrt{1 + z^2}}$

43. $\sqrt{1 - 9z^2}$ **45.** $\sqrt{2 + 2z + z^2}$

47. $\frac{1}{\sqrt{1 + 2z}}$ **49.** $\frac{\pi}{4}$

51. $\frac{\pi}{6}$ **53.** 0 **55.** $\sin^{-1}\frac{m}{r}$

57. $\tan^{-1}\frac{k}{h}$ **59.** $\tan^{-1}\frac{2}{x}$

61. $\sin^{-1}\frac{3{,}500}{z}$ **63.** $\cos^{-1}(-0.8)$

65. $\tan^{-1}4.1$ **67.** $\tan^{-1}\frac{5}{3}$

69. $\tan^{-1}50$ **71.** $\frac{\tan^{-1}9}{3}$

73. $\frac{2\sin^{-1}(-0.56)}{3}$ **75.** $\frac{\sin^{-1}0.75}{4}$

77. $\frac{\sin^{-1}\frac{25}{39}}{5}$ **79.** $\frac{\tan^{-1}\frac{D}{A} - C}{B}$

81. $\frac{\sin^{-1}0.6 - 3}{2}$

Solutions to trial exercise problems

11. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

We want the angle in quadrant I or quadrant IV whose tangent is $-\frac{\sqrt{3}}{3}$.

We know that $\tan 30° \left(\text{or } \frac{\pi}{6}\right)$ is $\frac{\sqrt{3}}{3}$.

Thus, the required values are $-30°$ or $-\frac{\pi}{6}$.

23. $\arctan(-0.2553)$

Calculator steps are the same in both degree and radian mode.

.2553 $\boxed{+/-}$ $\boxed{\text{INV}}$ $\boxed{\text{TAN}}$

$\boxed{\text{TI-81}}$ $\boxed{\text{TAN}^{-1}}$ $\boxed{(-)}$ .2553

$\boxed{\text{ENTER}}$

$(= -14.32168996°)$

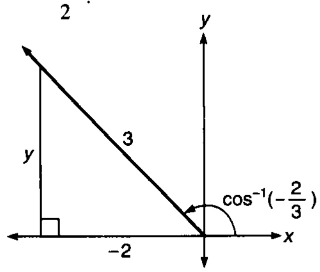
$(= -0.249960644)$

Thus, rounded, $\arctan(-0.2553) = -14.3°$ or -0.25 (radians).

31. $\tan[\cos^{-1}(-\frac{2}{3})]$

$\cos^{-1}(-\frac{2}{3})$ is an angle in quadrant II whose cosine is $-\frac{2}{3}$. Such an angle is shown in the reference triangle in the diagram. The Pythagorean theorem shows that $y = \sqrt{5}$, and we can then determine that the tangent is $\frac{\text{opp}}{\text{adj}}$

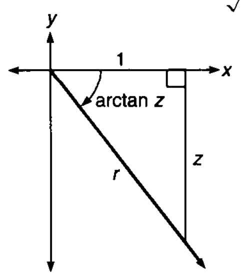
$= -\frac{\sqrt{5}}{2}$.



41. $\cos(\arctan z)$, $z < 0$

$\arctan z$ is an angle in quadrant IV if $z < 0$. A reference triangle is shown in the diagram. To find r: $r^2 = 1^2 + z^2$; $r = \sqrt{1 + z^2}$ (We know that $r > 0$.) Thus, using the reference triangle,

$\cos(\arctan z)$, $z < 0$, $= \frac{1}{\sqrt{1 + z^2}}$.

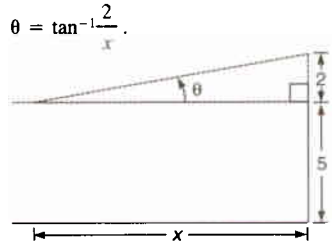


51. $\cos^{-1}\left(\cos\frac{11\pi}{6}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

59. Refer to the diagram.

$\tan\theta = \frac{2}{x}$, so

$\theta = \tan^{-1}\frac{2}{x}$.



77. $\frac{6\sin 5\theta}{5} = \frac{10}{13}$

$\frac{6\sin 5\theta}{5} = \frac{10}{13}$

Multiply by 5.

$6\sin 5\theta = \frac{50}{13}$

Multiply by $\frac{1}{6}$.

$\sin 5\theta = \frac{50}{78}$

Reduce.

$\sin 5\theta = \frac{25}{39}$

$5\theta = \sin^{-1}\frac{25}{39}$

Divide by 5.

$\theta = \frac{\sin^{-1}\frac{25}{39}}{5}$

81. $\sin(2x + 3) = 0.6$

$\sin(2x + 3) = 0.6$

$2x + 3 = \sin^{-1}0.6$

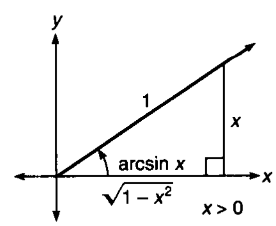
Subtract 3.

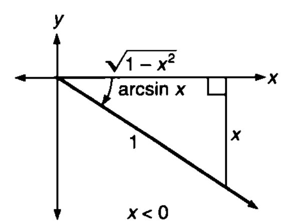
$2x = \sin^{-1}0.6 - 3$

Divide by 2.

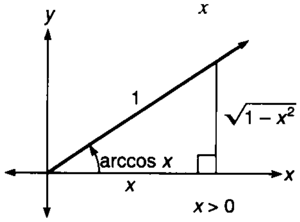
$x = \frac{\sin^{-1}0.6 - 3}{2}$

82. a. The two reference triangles in the first diagram show $\arcsin x$ when $x > 0$ and $x < 0$. In each case we can see that the tangent of this angle is $\frac{x}{\sqrt{1 - x^2}}$.

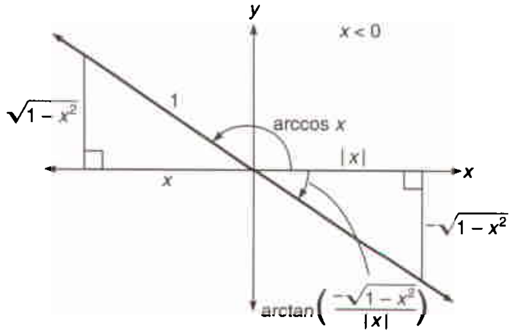




b. In the second diagram we see the reference triangle for arccos x, $x > 0$. It is easy to see that the tangent of the angle arccos x is $\dfrac{\sqrt{1-x^2}}{x}$.



In the third diagram we see both arctan $\dfrac{\sqrt{1-x^2}}{x}$, $x < 0$, and arccos x, $x < 0$. Note that $\dfrac{\sqrt{1-x^2}}{x}$, $x < 0 = \dfrac{-\sqrt{1-x^2}}{|x|}$, $x < 0$. We can see that they differ by π radians.



Exercise 4–4

Answers to odd-numbered problems

1. $\dfrac{\pi}{6}$ rad, 30° **3.** $\dfrac{\pi}{4}$ rad, 45°

5. $\dfrac{2\pi}{3}$ rad, 120° **7.** $\dfrac{\pi}{3}$ rad, 60°

9. $\dfrac{\pi}{2}$ rad, 90° **11.** 0.30 rad, 17.3°

13. 1.91, 109.5° **15.** 0.19 rad, 10.9°

17. 1.66 rad, 95.2° **19.** 0.32 rad, 18.3°

21. $\frac{1}{3}$ **23.** $\dfrac{\sqrt{15}}{15}$ **25.** $\sqrt{26}$

27. $\frac{3}{5}$ **29.** $-\dfrac{\sqrt{11}}{5}$ **31.** $\dfrac{1}{z}$

33. $\dfrac{\sqrt{z^2+1}}{z}$ **35.** $\dfrac{1}{2z}$ **37.** $\sqrt{z^2+2z}$

39. $\dfrac{3}{z}$

Solutions to trial exercise problems

7. arccsc $\dfrac{2\sqrt{3}}{3}$

By definition, arccsc $\dfrac{2\sqrt{3}}{3}$

$= \arcsin \dfrac{1}{\frac{2\sqrt{3}}{3}}$

$= \arcsin\left(\dfrac{3}{2\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}\right)$

$= \arcsin \dfrac{3\sqrt{3}}{2 \cdot 3}$

$= \arcsin \dfrac{\sqrt{3}}{2}$

$= 60°$ or $\dfrac{\pi}{3}$

17. $\sec^{-1}(-11.1261)$

$\sec^{-1}(-11.1261) = \cos^{-1}\left(\dfrac{1}{-11.1261}\right)$

by definition.

11.1261 $\boxed{+/-}$ $\boxed{1/x}$ $\boxed{\text{INV}}$

$\boxed{\text{COS}}$

$\boxed{\text{TI-81}}$ $\boxed{\text{COS}^{-1}}$ $\boxed{(-)}$ 11.1261

$\boxed{x^{-1}}$ $\boxed{\text{ENTER}}$

(= 95.15663191 in degree mode)

(= 1.660796532 in radian mode)

so $\sec^{-1}(-11.1261) = 95.2°$ or 1.66 (radians).

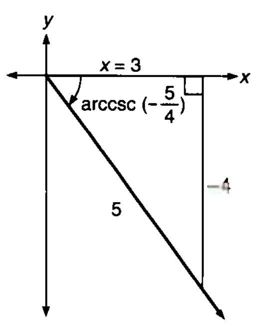
27. cos[arccsc($-\frac{5}{4}$)]

cos[arccsc($-\frac{5}{4}$)] = cos[arcsin($-\frac{4}{5}$)] by definition.

arcsin($-\frac{4}{5}$) is shown in the diagram.

The cosine of this angle is $\frac{3}{5}$. Thus,

cos[arccsc($-\frac{5}{4}$)] = $\frac{3}{5}$.



40. $\sec\left(\cot^{-1}\dfrac{2}{z+1}\right)$, $z + 1 > 0$

$\cot^{-1}\dfrac{2}{z+1}$, $z + 1 > 0$, is an angle in quadrant I; a reference triangle is shown in the diagram. We can find r.

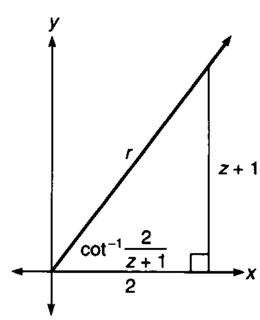
$r^2 = 2^2 + (z+1)^2$

$r^2 = z^2 + 2z + 5$

$r = \sqrt{z^2+2z+5}$

The cosine of the angle is

$\dfrac{2}{\sqrt{z^2+2z+5}}$, and so the secant, which is the reciprocal of the cosine, is $\dfrac{\sqrt{z^2+2z+5}}{2}$.



Chapter 4 review

1. Let $y = f(x) = 3x - 5$; show that $g(y) = x$.

$$g(y) = \frac{y+5}{3} = \frac{(3x-5)+5}{3} = x$$

Let $y = g(x) = \frac{x+5}{3}$; show that $f(y) = x$.

$$f(y) = 3y - 5 = 3\left(\frac{x+5}{3}\right) - 5 = x$$

2. Let $y = f(x) = \frac{3x}{x+1}$; show that $g(y) = x$.

$$g(y) = \frac{-y}{y-3} = \frac{-\frac{3x}{x+1}}{\frac{3x}{x+1} - 3}$$

$$= \frac{-\frac{3x}{x+1}}{\frac{3x}{x+1} - 3} \cdot \frac{x+1}{x+1}$$

$$= \frac{-3x}{3x - 3(x+1)} = x$$

Let $y = g(x) = \frac{-x}{x-3}$; show that $f(y) = x$.

$$f(y) = \frac{3y}{y+1} = \frac{3\frac{-x}{x-3}}{\frac{-x}{x-3} + 1}$$

$$= \frac{3\frac{-x}{x-3}}{\frac{-x}{x-3} + 1} \cdot \frac{x-3}{x-3}$$

$$= \frac{-3x}{-x + 1(x-3)} = x$$

3. no **4.** yes **5.** See figure 4–6.
6. $\text{domain}_{\sin^{-1}}\ 1 \le x \le 1$; $\text{range}_{\sin^{-1}}\ -\frac{\pi}{2} \le y \le \frac{\pi}{2}$ **7.** $-\frac{\pi}{6}$ rad, $-30°$
8. $\frac{\pi}{3}$ rad, $60°$ **9.** $\frac{\pi}{6}$ rad, $30°$
10. $\frac{-\pi}{4}$ rad, $-45°$ **11.** 0 rad, $0°$
12. $-\frac{\pi}{2}$ rad, $-90°$ **13.** 1.34 rad, $76.8°$
14. -0.51 rad, $-29.2°$ **15.** $\frac{4}{5}$
16. $\frac{\sqrt{15}}{15}$ **17.** $\frac{3\sqrt{5}}{5}$ **18.** $\frac{z\sqrt{1-z^2}}{1-z^2}$
19. $\sqrt{-z^2-2z}$ **20.** See figure 4–8.
21. $\text{domain}_{\tan^{-1}}\ R$; $\text{range}_{\tan^{-1}}\ -\frac{\pi}{2} < y < \frac{\pi}{2}$
22. $\frac{2\pi}{3}$ rad, $120°$
23. $\frac{\pi}{6}$ rad, $30°$ **24.** $\frac{\pi}{3}$ rad, $60°$
25. $-\frac{\pi}{4}$ rad, $-45°$ **26.** $\frac{\pi}{3}$ rad, $60°$
27. $-\frac{\pi}{4}$ rad, $-45°$ **28.** 1.00 rad, $57.3°$
29. 1.06 rad, $60.8°$ **30.** 1.88 rad, $107.8°$
31. $\frac{5\sqrt{34}}{34}$ **32.** 4 **33.** $-\frac{\sqrt{5}}{2}$
34. $\frac{2\pi}{3}$ rad, $120°$ **35.** $\frac{\sqrt{1-z^2}}{z}$
36. $\frac{1}{\sqrt{z^2+2z+2}}$ **37.** $\sin^{-1}\frac{j}{k}$
38. $\sin^{-1}\frac{7}{11}$ **39.** $\tan^{-1}\frac{35}{x}$
40. $\cos^{-1}0.89$ **41.** $\sin^{-1}(-0.88)$
42. $\sin^{-1}(-0.2)$ **43.** $\tan^{-1}5$
44. $\frac{\sin^{-1}0.76}{2}$ **45.** $\frac{\cos^{-1}0.7}{3}$
46. $\frac{\cos^{-1}(0.6) - 3}{2}$ **47.** $\frac{\sin^{-1}0.6}{2}$
48. $\frac{\pi}{3}$ rad, $60°$ **49.** $-\frac{\pi}{3}$ rad, $-60°$
50. $-\frac{\pi}{6}$ rad, $-30°$ **51.** $\frac{5\pi}{6}$ rad, $150°$
52. 0.23 rad, $13.2°$ **53.** 0.57 rad, $32.7°$
54. 1.82 rad, $104.3°$ **55.** -0.15 rad, $-8.6°$ **56.** $\frac{\sqrt{2}}{4}$ **57.** $\frac{\sqrt{15}}{15}$ **58.** $\frac{\sqrt{26}}{26}$
59. $\frac{7\sqrt{33}}{33}$ **60.** $\frac{\sqrt{z^2+1}}{z^2+1}$ **61.** $\sqrt{z^2-1}$
62. $\sqrt{z^2+2z}$ **63.** $\frac{1-z}{\sqrt{-2z+z^2}}$

Chapter 4 test

1. See figure 4–10. **2.** $\text{domain}_{\csc^{-1}}\ |x| \ge 1$; $\text{range}_{\csc^{-1}}\ \frac{-\pi}{2} \le y \le \frac{\pi}{2}$, $y \ne 0$
3. $\frac{2\pi}{3}$ rad, $120°$ **4.** $\frac{\pi}{4}$ rad, $45°$
5. $\frac{\pi}{6}$ rad, $30°$ **6.** $\frac{\pi}{3}$ rad, $60°$
7. 0.97 rad, $55.6°$ **8.** 2.00 rad, $114.6°$
9. 2.31 rad, $132.4°$ **10.** 0.29 rad, $16.6°$
11. $\frac{4}{3}$ **12.** $\frac{-4\sqrt{17}}{17}$ **13.** $-2\sqrt{2}$
14. $\frac{1}{z}$ **15.** $\frac{1}{\sqrt{z^2+2z+2}}$
16. $\sqrt{4z^2-1}$ **17.** $\frac{1-z}{\sqrt{z^2-2z}}$
18. $\sin^{-1}\frac{x}{z}$ **19.** $\sin^{-1}\frac{52}{x}$
20. $\sec^{-1}2.25$ **21.** $\sin^{-1}(-0.94)$
22. $\frac{\sin^{-1}0.75}{3}$ **23.** $\frac{3+\cos^{-1}0.6}{2}$

Chapter 5

Exercise 5-0

Solutions to all problems

1. $\sin\theta(\sin\theta - 1)$
2. $\cos\theta(\cos^2\theta + 3)$
3. $\cos^2\theta(\cos^2\theta - 1)$
$\cos^2\theta(\cos\theta - 1)(\cos\theta + 1)$
4. $\sin^3\theta(\sin^2\theta - 1)$
$\sin^3\theta(\sin\theta - 1)(\sin\theta + 1)$
5. $(\cos x - 4)(\cos x + 5)$
6. $(\tan x + 6)(\tan x - 4)$
7. $(2\sin x - 1)(\sin x - 3)$
8. $3\cos\theta(3\cos^2\theta - 5\cos\theta - 2)$
$3\cos\theta(3\cos\theta + 1)(\cos\theta - 2)$
9. $(2\csc\theta - 1)(3\csc\theta - 1)$
10. $\tan\theta(\tan\theta - 1) = 0$
$\tan\theta = 0$ or $\tan\theta - 1 = 0$
$\tan\theta = 0$ or $\tan\theta = 1$
11. $\tan^2\theta - \tan\theta - 2 = 0$
$(\tan\theta - 2)(\tan\theta + 1) = 0$
$\tan\theta - 2 = 0$ or $\tan\theta + 1 = 0$
$\tan\theta = 2$ or $\tan\theta = -1$
12. $(2\sin + 1)(3\sin\theta + 1) = 0$
$2\sin\theta + 1 = 0$ or $3\sin\theta + 1 = 0$
$2\sin\theta = -1$ or $3\sin\theta = -1$
$\sin\theta = -\frac{1}{2}$ or $\sin\theta = -\frac{1}{3}$
13. $(\sec^2\theta - 4)(\sec^2\theta - 1) = 0$
$(\sec\theta - 2)(\sec\theta + 2)(\sec\theta - 1)(\sec\theta + 1) = 0$
$\sec\theta - 2 = 0$ or $\sec\theta + 2 = 0$ or $\sec\theta - 1 = 0$ or $\sec\theta + 1 = 0$
$\sec\theta = \pm 2$ or ± 1
14. $(4\sin^2\theta - 1)(9\sin^2\theta - 1) = 0$
$(2\sin\theta - 1)(2\sin\theta + 1)$
$(3\sin\theta - 1)(3\sin\theta + 1) = 0$
$2\sin\theta - 1 = 0$ or $2\sin\theta + 1 = 0$ or $3\sin\theta - 1 = 0$ or $3\sin\theta + 1 = 0$
$2\sin\theta = 1$ or $2\sin\theta = -1$ or $3\sin\theta = 1$ or $3\sin\theta = -1$
$\sin\theta = \pm\frac{1}{2}$ or $\pm\frac{1}{3}$

15. $3\sin\theta\left(\frac{2}{3}\sin\theta\right) + 3\sin\theta\left(\frac{1}{3\sin\theta}\right)$

$= 1(3\sin\theta)$

$2\sin^2\theta + 1 = 3\sin\theta$

$2\sin^2\theta - 3\sin\theta + 1 = 0$

$(\sin\theta - 1)(2\sin\theta - 1) = 0$

$\sin\theta - 1 = 0$ or $2\sin\theta - 1 = 0$

$\sin\theta = 1$ or $2\sin\theta = 1$

$\sin\theta = 1$ or $\frac{1}{2}$

16. $\sin\theta = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-5)}}{2(1)}$

$\sin\theta = \dfrac{3 \pm \sqrt{29}}{2}$

17. $\sec\theta = \dfrac{-3 \pm \sqrt{3^2 - 4(2)(-7)}}{2(2)}$

$\sec\theta = \dfrac{-3 \pm \sqrt{65}}{4}$

18. $u = 2x - 6$

$u^3 - u^2 = 0$

$u^2(u - 1) = 0$

$u^2 = 0$ or $u - 1 = 0$

$u = 0$ or $u = 1$

$2x - 6 = 0$ or $2x - 6 = 1$

$2x = 6$ or $2x = 7$

$x = 3$ or $x = \frac{7}{2}$

19. $u = \dfrac{x}{3} - 1$

$12u^2 - 5u - 2 = 0$

$(3u - 2)(4u + 1) = 0$

$3u - 2 = 0$ or $4u + 1 = 0$

$u = \frac{2}{3}$ or $u = -\frac{1}{4}$

$\dfrac{x}{3} - 1 = \dfrac{2}{3}$ or $\dfrac{x}{3} - 1 = -\dfrac{1}{4}$

$x - 3 = 2$ or $x - 3 = -\frac{3}{4}$

$x = 5$ or $x = 2\frac{1}{4}$

20. $5\left(\dfrac{\pi}{2} - 3\right) + 3\left[\left(\dfrac{\pi}{2} - 3\right) - 7\right]$

$- \left(\dfrac{\pi}{2} - 3\right)$

$u = \dfrac{\pi}{2} - 3$

$5u + 3(u - 7) - u$

$5u + 3u - 21 - u$

$7u - 21$

$7\left(\dfrac{\pi}{2} - 3\right) - 21$

$\dfrac{7\pi}{2} - 21 - 21$

$\dfrac{7\pi}{2} - 42$

21. $u = 3x - \frac{1}{2}$

$\dfrac{2u^2 - 3u + 1}{u^2 - 1}$

$\dfrac{(u - 1)(2u - 1)}{(u - 1)(u + 1)}$

$\dfrac{2u - 1}{u + 1}$

$\dfrac{2(3x - \frac{1}{2}) - 1}{3x - \frac{1}{2} + 1}$

$\dfrac{6x - 1 - 1}{3x + \frac{1}{2}}$

$\dfrac{6x - 2}{3x + \frac{1}{2}} \cdot \dfrac{2}{2}$

$\dfrac{12x - 4}{6x + 1}$

Exercise 5–1

Answers to odd-numbered problems

1. $\dfrac{\sin\theta}{\frac{\sin\theta}{\cos\theta}}$

$\dfrac{\sin\theta\cos\theta}{\sin\theta}$

$\cos\theta$

3. $\dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{1}{\sin\theta}$

$\dfrac{1}{\cos\theta}$

$\sec\theta$

5. $\dfrac{\cos^2\theta}{\sin^2\theta}\sin^2\theta$

$\cos^2\theta$

7. $\sec^2\theta\cos^2\theta$

$\dfrac{1}{\cos^2\theta}\cos^2\theta$

1

9. $\dfrac{\sec^2\theta - 1}{\sin^2\theta}$

$\dfrac{\tan^2\theta}{\sin^2\theta}$

$\dfrac{\frac{\sin^2\theta}{\cos^2\theta}}{\sin^2\theta}$

$\dfrac{1}{\cos^2\theta}$

$\sec^2\theta$

11. $\dfrac{\frac{1}{\sin\theta}\sin\theta}{\cot\theta}$

$\dfrac{1}{\cot\theta}$

$\tan\theta$

13. $\cos\theta\sec\theta - \cos^2\theta$

$\cos\theta\dfrac{1}{\cos\theta} - \cos^2\theta$

$1 - \cos^2\theta$

$\sin^2\theta$

15. $\csc^2\theta\sin^2\theta$

$\dfrac{1}{\sin^2\theta}\sin^2\theta$

1

17. $\dfrac{\cos^2\theta}{\sin^2\theta} - \dfrac{1}{\sin^2\theta}$

$\dfrac{\cos^2\theta - 1}{\sin^2\theta}$

$\dfrac{-\sin^2\theta}{\sin^2\theta}$

-1

19. $\tan^2\theta\cot^2\theta + \tan^2\theta$

$\tan^2\theta \cdot \dfrac{1}{\tan^2\theta} + \tan^2\theta$

$1 + \tan^2\theta$

$\sec^2\theta$

21. $\sec^2\theta\cot^2\theta$

$\dfrac{1}{\cos^2\theta} \cdot \dfrac{\cos^2\theta}{\sin^2\theta}$

$\dfrac{1}{\sin^2\theta}$

$\csc^2\theta$

23. $\dfrac{\frac{\cos\theta}{\sin\theta}\,\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}$

$\dfrac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta}}$

1

25. $\sin x + \cos x\dfrac{\cos x}{\sin x}$

$\dfrac{\sin^2 x + \cos^2 x}{\sin x}$

$\dfrac{1}{\sin x}$

$\csc x$

27. $\dfrac{\sin x}{\cos x}\,\dfrac{1}{\sin x}\cos x$

1

29. $\dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x}\sin x$

$\dfrac{1 - \sin^2 x}{\cos x}$

$\dfrac{\cos^2 x}{\cos x}$

$\cos x$

31. $\csc x - \csc x\cos x + \cot x - \cot x\cos x$

$\dfrac{1}{\sin x} - \dfrac{1}{\sin x}\cos x + \dfrac{\cos x}{\sin x} - \dfrac{\cos x}{\sin x}\cos x$

$\dfrac{1 - \cos x + \cos x - \cos^2 x}{\sin x}$

$\dfrac{1 - \cos^2 x}{\sin x}$

$\dfrac{\sin^2 x}{\sin x}$

$\sin x$

33. $\csc\theta + \cot\theta$

$$\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$
$$\frac{1 + \cos\theta}{\sin\theta}$$

35. $\dfrac{\csc\theta}{\sec\theta + \tan\theta}$

$$\frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$$
$$\frac{\frac{1}{\sin\theta}}{\frac{1 + \sin\theta}{\cos\theta}}$$
$$\frac{\cos\theta}{\sin\theta + \sin^2\theta}$$

37. $\dfrac{1 + \csc\theta}{1 + \sec\theta}$

$$\frac{1 + \frac{1}{\sin\theta}}{1 + \frac{1}{\cos\theta}}$$
$$\frac{1 + \frac{1}{\sin\theta}}{1 + \frac{1}{\cos\theta}} \cdot \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$$
$$\frac{\sin\theta\cos\theta + \cos\theta}{\sin\theta\cos\theta + \sin\theta}$$
$$\frac{\cos\theta(\sin\theta + 1)}{\sin\theta(\cos\theta + 1)}$$
$$\frac{\cos\theta}{\sin\theta} \cdot \frac{\sin\theta + 1}{\cos\theta + 1}$$
$$\cot\theta\left(\frac{1 + \sin\theta}{1 + \cos\theta}\right)$$

39. $\dfrac{\tan^2\theta + \sec^2\theta}{\sec^2\theta}$

$$\frac{\tan^2\theta}{\sec^2\theta} + \frac{\sec^2\theta}{\sec^2\theta}$$
$$\tan^2\theta\cos^2\theta + 1$$
$$\frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta + 1$$
$$\sin^2\theta + 1$$

41. $\sec y - \cos y$

$$\frac{1}{\cos y} - \cos y$$
$$\frac{1}{\cos y} - \frac{\cos^2 y}{\cos y}$$
$$\frac{1 - \cos^2 y}{\cos y}$$
$$\frac{\sin^2 y}{\cos y}$$
$$\frac{\sin y}{\cos y}\sin y$$
$$\tan y \sin y$$

43. $\dfrac{1 + \cot^2\theta}{\tan^2\theta}$

$$\frac{1 + \frac{\cos^2\theta}{\sin^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}$$
$$\frac{\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}$$
$$\frac{\frac{1}{\sin^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}$$
$$\frac{\cos^2\theta}{\sin^4\theta}$$
$$\frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{1}{\sin^2\theta}$$
$$\cot^2\theta\csc^2\theta$$

45. $\dfrac{1}{\sec\theta - \cos\theta}$

$$\frac{1}{\frac{1}{\cos\theta} - \cos\theta}$$
$$\frac{1}{\frac{1 - \cos^2\theta}{\cos\theta}}$$
$$\frac{\cos\theta}{1 - \cos^2\theta}$$
$$\frac{\cos\theta}{\sin^2\theta}$$
$$\frac{\cos\theta}{\sin\theta}\,\frac{1}{\sin\theta}$$
$$\cot\theta\csc\theta$$

47. $\dfrac{\tan\theta}{\csc\theta}$

$$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\sin\theta}}$$
$$\frac{\sin^2\theta}{\cos\theta}$$
$$\sin\theta\,\frac{\sin\theta}{\cos\theta}$$
$$\sin\theta\tan\theta$$

49. $\dfrac{\tan^2\theta}{\sec\theta - 1}$

$$\frac{\sec^2\theta - 1}{\sec\theta - 1}$$
$$\frac{(\sec\theta + 1)(\sec\theta - 1)}{(\sec\theta - 1)}$$
$$\sec\theta + 1$$

51. $\dfrac{\cot x + 1}{\cot x - 1}$

$$\frac{\frac{\cos x}{\sin x} + 1}{\frac{\cos x}{\sin x} - 1}$$
$$\frac{\frac{\cos x + \sin x}{\sin x}}{\frac{\cos x - \sin x}{\sin x}}$$
$$\frac{\cos x + \sin x}{\cos x - \sin x}$$

53. $\dfrac{1 - \cos x}{\sin x}$

$$\frac{(1 - \cos x)(1 + \cos x)}{\sin x(1 + \cos x)}$$
$$\frac{1 - \cos^2 x}{\sin x(1 + \cos x)}$$
$$\frac{\sin^2 x}{\sin x(1 + \cos x)}$$
$$\frac{\sin x}{1 + \cos x}$$

55. $\dfrac{\cos x}{\sec x - \tan x}$

$$\frac{\cos x}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}}$$
$$\frac{\cos x}{\frac{1 - \sin x}{\cos x}}$$
$$\frac{\cos^2 x}{1 - \sin x}$$

57. $\dfrac{1}{1 - \sin x} + \dfrac{1}{1 + \sin x}$

$$\frac{1 + \sin x + 1 - \sin x}{(1 - \sin x)(1 + \sin x)}$$
$$\frac{2}{1 - \sin^2 x}$$
$$\frac{2}{\cos^2 x}$$
$$2\sec^2 x$$

59. $\sin^4 x - \cos^4 x$

$$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$$
$$(1)(1 - \cos^2 x - \cos^2 x)$$
$$1 - 2\cos^2 x$$

61. $\cot^2 x\cos^2 x$

$$\frac{\cos^2 x}{\sin^2 x}(1 - \sin^2 x)$$
$$\frac{\cos^2 x}{\sin^2 x} - \cos^2 x$$
$$\cot^2 x - \cos^2 x$$

63. $\dfrac{\tan y - \cot y}{\tan y + \cot y}$

$$\frac{\tan y - \frac{1}{\tan y}}{\tan y + \frac{1}{\tan y}}$$
$$\frac{\frac{\tan^2 y - 1}{\tan y}}{\frac{\tan^2 y + 1}{\tan y}}$$
$$\frac{\tan^2 y - 1}{\tan^2 y + 1}$$
$$\frac{\tan^2 y - 1}{\sec^2 y}$$

65. $\dfrac{1 - \sin x}{1 + \sin x}$

$$\frac{(1 - \sin x)(1 - \sin x)}{(1 + \sin x)(1 - \sin x)}$$
$$\frac{1 - 2\sin x + \sin^2 x}{1 - \sin^2 x}$$
$$\frac{1 - 2\sin x + \sin^2 x}{\cos^2 x}$$
$$\frac{1}{\cos^2 x} - 2\left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\cos x}\right) + \frac{\sin^2 x}{\cos^2 x}$$
$$\sec^2 x - 2\tan x\sec x + \tan^2 x$$
$$(\tan x - \sec x)^2$$

67. $\sec^4 x - \sec^2 x$

$$\sec^2 x(\sec^2 x - 1)$$
$$(1 + \tan^2 x)(\tan^2 x)$$
$$\tan^2 x + \tan^4 x$$

69. $2\cos^2 y - 1$

$$\cos^2 y + \cos^2 y - 1$$
$$\cos^2 y - (1 - \cos^2 y)$$
$$\cos^2 y - \sin^2 y$$

71. Let $\theta = \frac{\pi}{6}$:

$1 - \cos\theta = 1 - \cos\frac{\pi}{6}$

$= 1 - \frac{\sqrt{3}}{2}$

$= \frac{2-\sqrt{3}}{2}$

$\sin\frac{\pi}{6} = \frac{1}{2} \neq \frac{2-\sqrt{3}}{2}$

73. Let $\theta = \frac{\pi}{4}$;

$\sec\frac{\pi}{4} \stackrel{?}{=} \frac{1}{\csc\frac{\pi}{4}}$

$\sqrt{2} \neq \frac{1}{\sqrt{2}}$

75. Let $\theta = \frac{\pi}{2}$:

$\sin^2\theta - 2\cos\theta\sin\theta + \cos^2\theta$

$\sin^2\frac{\pi}{2} - 2\cos\frac{\pi}{2}\sin\frac{\pi}{2} + \cos^2\frac{\pi}{2}$

$1 - 2(0)(1) + 0 = 1$

$1 \neq 2$

77. $\csc\theta + \sec\theta\cot\theta$

$\csc\frac{\pi}{6} + \sec\frac{\pi}{6}\cot\frac{\pi}{6}$

$2 + \left(\frac{2\sqrt{3}}{3}\right)\left(\frac{3\sqrt{3}}{3}\right)$

$2 + 2 = 4$

$4 \neq 2$

79. $\frac{1-\cos\theta}{1+\cos\theta}$

$\frac{1-\cos\frac{\pi}{4}}{1+\cos\frac{\pi}{4}} = \frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}} = \frac{2-\sqrt{2}}{2+\sqrt{2}}$

$\sin^2\frac{\pi}{4} = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$

$\frac{2-\sqrt{2}}{2+\sqrt{2}} \neq \frac{1}{2}$

81. a. $\left(1 - \csc^2\frac{\pi}{6}\right)\left(1 - \sec^2\frac{\pi}{6}\right)$

$(1 - 2^2)\left[1 - \left(\frac{2}{\sqrt{3}}\right)^2\right]$

$(-3)(-\frac{1}{3}) = 1$

b. $\left(1 - \csc^2\frac{\pi}{4}\right)\left(1 - \sec^2\frac{\pi}{4}\right)$

$[1 - (\sqrt{2})^2][1 - (\sqrt{2})^2]$

$(1-2)(1-2) = 1$

c. yes

83. a. $2\sin^2\frac{\pi}{6} + \sin\frac{\pi}{6}$

$2\left(\frac{1}{2}\right)^2 + \frac{1}{2} = 1$

b. $2\sin^2\frac{3\pi}{2} + \sin\frac{3\pi}{2}$

$2(-1)^2 + (-1) = 1$

c. no; $\theta = \frac{\pi}{4}$ is a counterexample

Solutions to trial exercise problems

18. $\tan^2\theta - \sec^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$

$\frac{\sin^2\theta - 1}{\cos^2\theta}$

$\frac{-(1-\sin^2\theta)}{\cos^2\theta}$

$\frac{-\cos^2\theta}{\cos^2\theta}$

-1

32. $\frac{\sec^4 y - \tan^4 y}{\sec^2 y + \tan^2 y}$

$\frac{(\sec^2 y + \tan^2 y)(\sec^2 y - \tan^2 y)}{\sec^2 y + \tan^2 y}$

$\sec^2 y - \tan^2 y$

$(\tan^2 y + 1) - \tan^2 y$

1

36. $\frac{\sec\theta}{\csc\theta + \cot\theta}$

$\frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}}$

$\frac{\frac{1}{\cos\theta}}{\frac{1+\cos\theta}{\sin\theta}}$

$\frac{1}{\cos\theta}\cdot\frac{\sin\theta}{1+\cos\theta}$

$\tan\theta\,\frac{1}{1+\cos\theta}$

$\frac{\tan\theta}{1+\cos\theta}$

52. $\frac{\csc y + 1}{\csc y - 1}$

$\frac{\frac{1}{\sin y} + 1}{\frac{1}{\sin y} - 1}$

$\frac{\frac{1+\sin y}{\sin y}}{\frac{1-\sin y}{\sin y}}$

$\frac{\frac{1+\sin y}{\sin y}}{\frac{1-\sin y}{\sin y}} \cdot \frac{\sin y}{\sin y}$

$\frac{1+\sin y}{1-\sin y}$

84. a. $\tan^4\theta - \tan^2\theta = 6$

$\theta = \frac{\pi}{3}$: $(\sqrt{3})^4 - (\sqrt{3})^2 = 6$

$9 - 3 = 6$

b. $\theta = \frac{4\pi}{3}$: $(\sqrt{3})^4 - (\sqrt{3})^2 = 6$

$9 - 3 = 6$

c. no;

let $\theta = 0$: $0^4 - 0^2 \neq 6$

Exercise 5–2

Answers to odd-numbered problems

1. $\cos 72°$ **3.** $\cot 82°$ **5.** $\csc\frac{\pi}{6}$

7. $\sin\left(-\frac{\pi}{3}\right)$ **9.** $\csc\frac{5\pi}{4}$ **11.** 1

13. 1 **15.** 1 **17.** -1 **19.** 1

21. 1 **23.** 1 **25.** 1

27. $\frac{\sqrt{6}+\sqrt{2}}{4}$ **29.** $\frac{\sqrt{6}+\sqrt{2}}{4}$

31. $\frac{\sqrt{6}+\sqrt{2}}{4}$ **33.** $\frac{\sqrt{6}-\sqrt{2}}{4}$

35. $\frac{\sqrt{2}-\sqrt{6}}{4}$ **37.** $2 + \sqrt{3}$

39. $\frac{2\sqrt{14}+3}{12}$ **41.** $\frac{-15-12\sqrt{7}}{36-5\sqrt{7}}$

43. $\frac{13}{85}$ **45.** $\frac{-\sqrt{5}-4\sqrt{2}}{9}$

47. $\frac{(8+\sqrt{5})\sqrt{17}}{51}$ **49.** $-\frac{119}{120}$

51. $-\frac{\sqrt{5}}{5}$ **53.** $\frac{-4\sqrt{6}+\sqrt{5}}{15}$

55. $2 + \sqrt{3}$

57. $\sin(\pi - \theta) = \sin\pi\cos\theta - \cos\pi\sin\theta$
$= 0\cos\theta - (-1)\sin\theta$
$= \sin\theta$

59. $\cos(\pi - \theta) = \cos\pi\cos\theta + \sin\pi\sin\theta$
$= (-1)\cos\theta + 0\sin\theta$
$= -\cos\theta$

61. $\tan(\pi - \theta) = \frac{\tan\pi - \tan\theta}{1 + \tan\pi\tan\theta}$
$= \frac{0 - \tan\theta}{1 + 0\tan\theta}$
$= -\tan\theta$

63. $\sin(\theta + 2\pi) = \sin\theta\cos 2\pi + \cos\theta\sin 2\pi$
$= \sin\theta(1) + \cos\theta(0)$
$= \sin\theta$

65. $\tan(\theta + \pi) = \frac{\tan\theta + \tan\pi}{1 - \tan\theta\tan\pi}$
$= \frac{\tan\theta + 0}{1 - \tan\theta(0)}$
$= \tan\theta$

67. $\frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$
$\frac{1}{2}[\sin \alpha \cos \beta + \cos \alpha \sin \beta - (\sin \alpha \cos \beta - \cos \alpha \sin \beta)]$
$\frac{1}{2}[2 \cos \alpha \sin \beta]$
$\cos \alpha \sin \beta$

69. $\frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\frac{1}{2}[\cos \alpha \cos \beta + \sin \alpha \sin \beta - (\cos \alpha \cos \beta - \sin \alpha \sin \beta)]$
$\frac{1}{2}[2 \sin \alpha \sin \beta]$
$\sin \alpha \sin \beta$

71. **a.** $\frac{11}{29}$ **b.** $3\frac{12}{29}$

73. $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$
This was shown true in the text.
$\cos(\alpha + (-\beta)) = \cos \alpha \cos(-\beta) - \sin \alpha \sin(-\beta)$
Replace β by $-\beta$. This is valid since the identity is true for all angles and α.
$\cos(\alpha - \beta) = \cos \alpha \cos \beta - \sin \alpha[-\sin \beta]$
$\alpha + (-\beta) = \alpha - \beta$; $\cos(-\theta) = \cos \theta$; $\sin(-\theta) = -\sin \theta$.
$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$
This statement is true since the preceding statements are true.

75. $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$
Proved true above.
Let $\alpha = \frac{\pi}{2} - \theta$. Then $\theta = \frac{\pi}{2} - \alpha$.
$\cos \alpha = \sin\left(\frac{\pi}{2} - \alpha\right)$
Substitution of expression (section 5–0).
$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$
The variable name α or θ is unimportant.

77. $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$
Identity [3].
$\sin(\alpha + (-\beta)) = \sin \alpha \cos(-\beta) + \cos \alpha \sin(-\beta)$
$\sin(\alpha - \beta) = \sin \alpha \cos \beta + \cos \alpha[-\sin \beta]$
Cosine is an even function, sine is odd.
$= \sin \alpha \cos \beta - \cos \alpha \sin \beta$
This is identity [4].

79. $\cot\left(\frac{\pi}{2} - \theta\right) = \dfrac{1}{\tan\left(\frac{\pi}{2} - \theta\right)} = \dfrac{1}{\cot \theta} = \tan \theta.$

81. $\csc\left(\frac{\pi}{2} - \theta\right) = \dfrac{1}{\sin\left(\frac{\pi}{2} - \theta\right)} = \dfrac{1}{\cos \theta} = \sec \theta.$

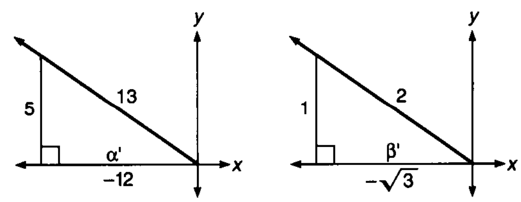
Solutions to trial exercise problems

8. $\sin\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{2} - \left(-\frac{\pi}{3}\right)\right) = \cos \frac{5\pi}{6}$

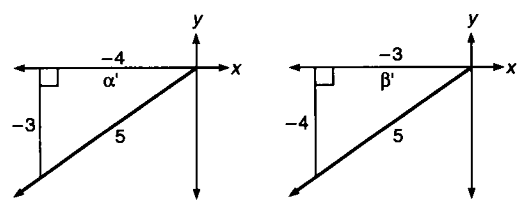
17. $\tan^2 8^\circ - \csc^2 82^\circ$
$\tan^2 8^\circ - \sec^2(90^\circ - 82^\circ)$
$\tan^2 8^\circ - \sec^2 8^\circ$
$-(\sec^2 8^\circ - \tan^2 8^\circ) = -1$

28. $\tan \frac{\pi}{12}$
$\tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$
$$\frac{\tan \frac{\pi}{4} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{4} \tan \frac{\pi}{6}}$$
$$\frac{1 - \frac{\sqrt{3}}{3}}{1 + 1\left(\frac{\sqrt{3}}{3}\right)} \cdot \frac{3}{3} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$$
$$\frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3}$$

40. $\cos \alpha = -\frac{12}{13}$, quadrant II; $\sin \beta = \frac{1}{2}$, quadrant II.
Find $\cos(\alpha - \beta)$.
$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$
Find $\cos \alpha$, $\sin \alpha$, $\cos \beta$, $\sin \beta$ from the reference triangles in the figure.
$$= -\frac{12}{13} \cdot \left(-\frac{\sqrt{3}}{2}\right) + \frac{5}{13} \cdot \frac{1}{2} = \frac{12\sqrt{3} + 5}{26}.$$



48. $\tan \alpha = \frac{3}{4}$, quadrant III; $\sin \beta = -\frac{4}{5}$, quadrant III. Find $\cos(\alpha - \beta)$.
$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$
$= -\frac{4}{5}(-\frac{3}{5}) + (-\frac{3}{5})(-\frac{4}{5}) = \frac{24}{25}.$



70. For angle α, the hypotenuse is $\sqrt{164} = 2\sqrt{41}$, and for angle β, the hypotenuse is $\sqrt{136} = 2\sqrt{34}$. Thus, $\sin\alpha = \dfrac{8}{2\sqrt{41}}$ $= \dfrac{4}{\sqrt{41}}$; $\cos\alpha = \dfrac{10}{2\sqrt{41}} = \dfrac{5}{\sqrt{41}}$; $\sin\beta = \dfrac{6}{2\sqrt{34}} = \dfrac{3}{\sqrt{34}}$;
$\cos\beta = \dfrac{10}{2\sqrt{34}} = \dfrac{5}{\sqrt{34}}$
Therefore, $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$= \left(\frac{4}{\sqrt{41}}\right)\left(\frac{5}{\sqrt{34}}\right) - \left(\frac{5}{\sqrt{41}}\right)\left(\frac{3}{\sqrt{34}}\right)$$
$$= \frac{20}{\sqrt{1{,}394}} - \frac{15}{\sqrt{1{,}394}} = \frac{5}{\sqrt{1{,}394}}$$
$$= \frac{5\sqrt{1{,}394}}{1{,}394}$$

76. $\sin\theta = \cos\left(\dfrac{\pi}{2} - \theta\right)$
We know this is true.
Replace θ by $\alpha + \beta$.
$\sin(\alpha + \beta) = \cos\left[\dfrac{\pi}{2} - (\alpha + \beta)\right]$
$\sin(\alpha + \beta) = \cos\left[\left(\dfrac{\pi}{2} - \alpha\right) - \beta\right]$
Regroup $\dfrac{\pi}{2} - \alpha - \beta$.
$= \cos\left(\dfrac{\pi}{2} - \alpha\right)\cos\beta + \sin\left(\dfrac{\pi}{2} - \alpha\right)\sin\beta$
Use the identity for $\cos(\alpha - \beta)$.
$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$
Use cofunction identities.
Thus, $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.

Exercise 5–3

Answers to odd-numbered problems

1. $\sin\dfrac{\pi}{2}$ **3.** $\cos 6\pi$ **5.** $\cos\dfrac{\pi}{5}$

7. $3\tan 20°$ **9.** $\sin 12\theta$ **11.** $3\cos 10\theta$

13. $5\tan 6\theta$ **15.** $2\cos 14\theta$

17. $3\cos 6\theta$ **19.** $70°$ **21.** $\dfrac{5\pi}{12}$

23. $35°$ **25.** $20°$ **27.** $\dfrac{\pi}{8}$ **29.** $\dfrac{4\pi}{5}$

31. $\frac{24}{25}, \frac{7}{25}, \frac{24}{7}$ **33.** $-\frac{24}{25}, \frac{7}{25}, -\frac{24}{7}$

35. $\dfrac{5\sqrt{39}}{32}, \dfrac{7}{32}, \dfrac{5\sqrt{39}}{7}$

37. $\dfrac{\sqrt{70}}{10}, -\dfrac{\sqrt{30}}{10}, -\dfrac{\sqrt{21}}{3}$

39. $\dfrac{\sqrt{50 - 20\sqrt{5}}}{10}, -\dfrac{\sqrt{50 + 20\sqrt{5}}}{10}, 2 - \sqrt{5}$

41. $\dfrac{\sqrt{2 - \sqrt{3}}}{2}, \dfrac{\sqrt{2 + \sqrt{3}}}{2}, 2 - \sqrt{3}$

43. $\frac{1}{2}\sqrt{2 + \sqrt{3}}, \frac{1}{2}\sqrt{2 - \sqrt{3}}, 2 + \sqrt{3}$

45. $\dfrac{\sqrt{4 + \sqrt{6} + 2\sqrt{3} + 2\sqrt{2}} - \sqrt{4 + \sqrt{6} - 2\sqrt{3} - 2\sqrt{2}}}{4}$

47. $\dfrac{\sqrt{2 - \sqrt{2 + \sqrt{3}}}}{2}$

49. $(\sin\theta + \cos\theta)^2$
$\sin^2 + 2\sin\theta\cos\theta + \cos^2\theta$
$2\sin\theta\cos\theta + 1$
$\sin 2\theta + 1$

51. $\cos^4\theta - \sin^4\theta$
$(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$
$(1)(\cos 2\theta)$
$\cos 2\theta$

53. $\dfrac{(\cos^2\theta + \sin^2\theta) + (\cos^2\theta - \sin^2\theta)}{(\cos^2\theta + \sin^2\theta) - (\cos^2\theta - \sin^2\theta)}$
$\dfrac{2\cos^2\theta}{2\sin^2\theta}$
$\cot^2\theta$

55. $\dfrac{2\cos 2\theta}{\sin 2\theta}$
$\dfrac{2(\cos^2\theta - \sin^2\theta)}{2\sin\theta\cos\theta}$
$\dfrac{\cos^2\theta}{\sin\theta\cos\theta} - \dfrac{\sin^2\theta}{\sin\theta\cos\theta}$
$\dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}$
$\cot\theta - \tan\theta$

57. $\sin 2\theta\cos 2\theta$
$2\sin\theta\cos\theta(\cos^2\theta - \sin^2\theta)$
$2\sin\theta\cos\theta(\cos^2\theta) - 2\sin^3\theta\cos\theta$
$2\sin\theta\cos\theta(1 - \sin^2\theta) - 2\sin^3\theta\cos\theta$
$2\sin\theta\cos\theta - 2\sin^3\theta\cos\theta - 2\sin^3\theta\cos\theta$
$2\sin\theta\cos\theta - 4\sin^3\theta\cos\theta$

59. $\dfrac{2}{1 - \cos 2\theta}$
$\dfrac{2}{1 - (\cos^2\theta - \sin^2\theta)}$
$\dfrac{2}{1 - \cos^2\theta + \sin^2\theta}$
$\dfrac{2}{\sin^2\theta + \sin^2\theta}$
$\dfrac{2}{2\sin^2\theta}$
$\dfrac{1}{\sin^2\theta}$
$\csc^2\theta$

61. $\dfrac{2(\tan\theta + \tan^3\theta)}{1 - \tan^4\theta}$
$\dfrac{2\tan\theta(1 + \tan^2\theta)}{(1 - \tan^2\theta)(1 + \tan^2\theta)}$
$\dfrac{2\tan\theta}{1 - \tan^2\theta}$
$\tan 2\theta$

63. $2\csc 2\theta\sin\theta\cos\theta$
$2\left(\dfrac{1}{\sin 2\theta}\right)\sin\theta\cos\theta$
$\dfrac{2\sin\theta\cos\theta}{\sin 2\theta}$
$\dfrac{2\sin\theta\cos\theta}{2\sin\theta\cos\theta}$
1

65. $\dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$
$\dfrac{1 - \dfrac{\sin^2\theta}{\cos^2\theta}}{1 + \dfrac{\sin^2\theta}{\cos^2\theta}}$
$\dfrac{\dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}}$
$\dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$
$\dfrac{\cos^2\theta - \sin^2\theta}{1}$
$\cos^2\theta - \sin^2\theta$
$\cos 2\theta$

67. $\cos^2\frac{\theta}{2}$

$\left(\pm\sqrt{\frac{1+\cos\theta}{2}}\right)^2$

$\frac{1+\cos\theta}{2}$

$\frac{1+\cos\theta}{2}\cdot\frac{1-\cos\theta}{1-\cos\theta}$

$\frac{1-\cos^2\theta}{2-2\cos\theta}$

69. $\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2}$

$\left(\pm\sqrt{\frac{1+\cos\theta}{2}}\right)^2\left(\pm\sqrt{\frac{1-\cos\theta}{2}}\right)^2$

$\left(\pm\sqrt{\frac{1-\cos^2\theta}{4}}\right)^2$

$\frac{\sin^2\theta}{4}$

71. $\tan^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}$

$\left(\pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right)^2+\left(\pm\sqrt{\frac{1+\cos\theta}{2}}\right)^2$

$\frac{1-\cos\theta}{1+\cos\theta}+\frac{1+\cos\theta}{2}$

$\frac{2-2\cos\theta+1+2\cos\theta+\cos^2\theta}{2(1+\cos\theta)}$

$\frac{\cos^2\theta+3}{2+2\cos\theta}$

73. $\frac{\csc\theta-\cot\theta}{2\csc\theta}$

$\frac{\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}}{\frac{2}{\sin\theta}}$

$\frac{\frac{1-\cos\theta}{\sin\theta}}{\frac{2}{\sin\theta}}$

$\frac{1-\cos\theta}{2}$

$\left(\pm\sqrt{\frac{1-\cos\theta}{2}}\right)^2$

$\sin^2\frac{\theta}{2}$

75. $\frac{1+\sec\theta}{\sec\theta}$

$\frac{1+\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$

$\frac{\frac{\cos\theta+1}{\cos\theta}}{\frac{1}{\cos\theta}}$

$\cos\theta+1$

$2\left(\frac{1+\cos\theta}{2}\right)$

$2\left(\pm\sqrt{\frac{1+\cos\theta}{2}}\right)^2$

$2\cos^2\frac{\theta}{2}$

77. $2\cos^2\frac{\theta}{2}-\cos\theta$

$2\left(\pm\sqrt{\frac{1+\cos\theta}{2}}\right)^2-\cos\theta$

$1+\cos\theta-\cos\theta$

1

79. $\cos 3\theta=\cos(2\theta+\theta)$

$=\cos(2\theta)\cos\theta-\sin(2\theta)\sin\theta$

$(\cos^2\theta-\sin^2\theta)\cos\theta-2\sin\theta\cos\theta\sin\theta$

$\cos^3\theta-\sin^2\theta\cos\theta-2\sin^2\theta\cos\theta$

$\cos^3\theta-3\sin^2\theta\cos\theta$

$\cos^2\theta-3(1-\cos^2\theta)\cos\theta$

$\cos^3\theta-3\cos\theta+3\cos^3\theta$

$4\cos^3\theta-3\cos\theta$

81. We know that $\sin 4\theta=4\cos^3\theta\sin\theta-4\sin^3\theta\cos\theta$ and $\cos 4\theta=8\cos^4\theta-8\cos^2\theta+1$ from problem 80.

a. $\sin 5\theta$

$=\sin(\theta+4\theta)$

$=\sin\theta\cos 4\theta+\cos\theta\sin 4\theta$

$=\sin\theta(8\cos^4\theta-8\cos^2\theta+1)+\cos\theta(4\sin\theta\cos\theta-8\sin^3\theta\cos\theta)$

$=4\sin\theta\cos^2\theta-8\sin^3\theta\cos^2\theta+8\sin\theta\cos^4\theta-8\sin\theta\cos^2\theta+\sin\theta$

We know $\cos^2\theta=1-\sin^2\theta$, so that $\cos^4\theta=(1-\sin^2\theta)^2=1-2\sin^2\theta+\sin^4\theta$. Replace $\cos^2\theta$ and $\cos^4\theta$ in the equation above:

$=4\sin\theta(1-\sin^2\theta)-8\sin^3\theta(1-\sin^2\theta)+8\sin\theta(1-2\sin^2\theta+\sin^4\theta)-8\sin\theta(1-\sin^2\theta)+\sin\theta$

$=4\sin\theta-4\sin^3\theta-8\sin^3\theta+8\sin^5\theta+8\sin\theta-16\sin^3\theta+8\sin^5\theta-8\sin\theta+8\sin^3\theta+\sin\theta$

$=16\sin^5\theta-20\sin^3\theta+5\sin\theta$

b. $\cos 5\theta$

$=\cos(\theta+4\theta)$

$=\cos\theta\cos 4\theta-\sin\theta\sin 4\theta$

$=\cos\theta(8\cos^4\theta-8\cos^2\theta+1)-\sin\theta(4\cos^3\theta\sin\theta-4\sin^3\theta\cos\theta)$

$=8\cos^5\theta-8\cos^3\theta+\cos\theta-4\cos^3\theta\sin^2\theta+4\sin^4\theta\cos\theta$

$=8\cos^5\theta-8\cos^3\theta+\cos\theta-4\cos^3\theta(1-\cos^2\theta)+4(1-\cos^2\theta)^2\cos\theta$

$=8\cos^5\theta-8\cos^3\theta+\cos\theta-4\cos^3\theta+4\cos^5\theta+4\cos\theta-8\cos^3\theta+4\cos^5\theta$

$=16\cos^5\theta-20\cos^3\theta+5\cos\theta$

83. $\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha}$; let $\frac{\alpha}{2} = \theta$, so $\alpha = 2\theta$.

$$\tan\theta = \frac{1-\cos 2\theta}{\sin 2\theta} = \frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta} = \frac{2\sin^2\theta}{2\sin\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

If every instance of θ above were replaced by $\frac{\alpha}{2}$, then the statements would still be true, thus proving the identity.

85. a. Problem 41 shows that $\sin 15° = \frac{\sqrt{2-\sqrt{3}}}{2}$.

b. $\sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

c. $\frac{\sqrt{6}-\sqrt{2}}{4} \overset{?}{=} \frac{\sqrt{2-\sqrt{3}}}{2}$

$\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)^2 \quad \left(\frac{\sqrt{2-\sqrt{3}}}{2}\right)^2$

Square both values.

$\left(\frac{(\sqrt{6}-\sqrt{2})^2}{16}\right) \quad \frac{2-\sqrt{3}}{4}$

$\frac{6-2\sqrt{12}+2}{16}$

$\frac{8-4\sqrt{3}}{16}$

$\frac{4(2-\sqrt{3})}{16}$

$\frac{2-\sqrt{3}}{4}$

87. Identity [5]

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta + 1 = 2\cos^2\theta$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\cos\theta = \pm\sqrt{\frac{1+\cos 2\theta}{2}}$

$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

Identity [6-a]

$$\tan\frac{\alpha}{2} = \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{\pm\sqrt{\frac{1-\cos\alpha}{2}}}{\pm\sqrt{\frac{1+\cos\alpha}{2}}} = \pm\sqrt{\frac{\frac{1-\cos\alpha}{2}}{\frac{1+\cos\alpha}{2}}} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

89. $x = 13\frac{37}{47}$

91. $\sin 2\alpha + \sin 2\beta = 2\sin(\alpha+\beta)\cdot\cos(\alpha-\beta)$

$2\sin\alpha\cos\alpha + 2\sin\beta\cos\beta$

$= 2(\sin\alpha\cos\beta + \cos\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$= 2(\sin\alpha\cos^2\beta\cos\alpha + \sin^2\alpha\cos\beta\sin\beta$
$\quad + \cos^2\alpha\sin\beta\cos\beta + \cos\alpha\sin^2\beta\sin\alpha)$

$= 2[\sin\alpha\cos^2\beta\cos\alpha + \cos\alpha\sin^2\beta\sin\alpha$
$\quad + \cos^2\alpha\sin\beta\cos\beta + \sin^2\alpha\cos\beta\sin\beta]$

$= 2[\sin\alpha\cos\alpha(\cos^2\beta + \sin^2\beta)$
$\quad + \sin\beta\cos\beta(\cos^2\alpha + \sin^2\alpha)]$

$= 2[\sin\alpha\cos\alpha(1) + \sin\beta\cos\beta(1)]$

$= 2\sin\alpha\cos\alpha + 2\sin\beta\cos\beta$

93. $\cos 2\alpha + \cos 2\beta = 2\cos(\alpha+\beta)\cdot\cos(\alpha-\beta)$

$(2\cos^2\alpha - 1) + (2\cos^2\beta - 1)$

$= 2(\cos\alpha\cos\beta - \sin\alpha\sin\beta)(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$2\cos^2\alpha + 2\cos^2\beta - 2$

$= 2(\cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta)$

$= 2[\cos^2\alpha\cos^2\beta - (1-\cos^2\alpha)(1-\cos^2\beta)]$

$= 2[\cos^2\alpha\cos^2\beta - (1 - \cos^2\beta - \cos^2\alpha + \cos^2\alpha\cos^2\beta)]$

$= 2[-1 + \cos^2\beta + \cos^2\alpha]$

$= 2\cos^2\alpha + 2\cos^2\beta - 2$

95.
$$\cot 2\theta = \frac{\cos 2\theta}{\sin 2\theta} = \frac{\cos^2\theta - \sin^2\theta}{2\sin\theta\cos\theta} = \frac{1}{2}\cdot\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta} = \frac{1}{2}\left(\frac{\cos^2\theta}{\sin\theta\cos\theta} - \frac{\sin^2\theta}{\sin\theta\cos\theta}\right) = \frac{1}{2}\left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right) = \frac{1}{2}(\cot\theta - \tan\theta) = \frac{1}{2}\left(\cot\theta - \frac{1}{\cot\theta}\right)$$

Solutions to trial exercise problems

6. $\dfrac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}}$

$\frac{2\tan\alpha}{1-\tan^2\alpha} = \tan 2\alpha$; Let $\alpha = \frac{\pi}{6}$, so $2\alpha = \frac{\pi}{3}$,

and $\tan 2\alpha$ becomes $\tan\frac{\pi}{3}$.

8. $8\cos^2\frac{\pi}{2} - 4$

$2\cos^2\alpha - 1 = \cos 2\alpha$

$8\cos^2\alpha - 1 = 4\cos 2\alpha$

Multiply each member by 4.

Let $\alpha = \frac{\pi}{2}$, so $2\alpha = \pi$, and $4\cos 2\alpha = 4\cos\pi$.

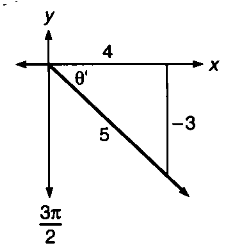
27. $\cos\theta = \sqrt{\frac{1}{2}\left(1 + \cos\frac{\pi}{4}\right)}$

$\cos\frac{\alpha}{2} = \sqrt{\frac{1 + \cos\alpha}{2}}$

$\alpha = \frac{\pi}{4}$, so $\frac{\alpha}{2} = \frac{\pi}{8}$. Thus, θ is $\frac{\pi}{8}$, and

$\cos\frac{\pi}{8} = \sqrt{\frac{1}{2}\left(1 + \cos\frac{\pi}{4}\right)}$.

34. $\tan\theta = -\frac{3}{4}, \frac{3\pi}{2} < \theta < 2\pi$

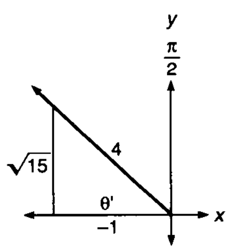


$\sin 2\theta = 2\sin\theta\cos\theta = 2(-\frac{3}{5})\cdot\frac{4}{5} = -\frac{24}{25}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = (\frac{4}{5})^2 - (-\frac{3}{5})^2 = \frac{7}{25}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = -\frac{24}{7}$

38. $\tan\theta = -\sqrt{15}, \frac{\pi}{2} < \theta < \pi$: $\cos\theta = -\frac{1}{4}$



$\frac{\pi}{4} \le \frac{\theta}{2} \le \frac{\pi}{2}$, $\left(\frac{\theta}{2} \text{ in quadrant I}\right)$ so

$\sin\frac{\theta}{2} > 0$, $\cos\frac{\theta}{2} > 0$, $\tan\frac{\theta}{2} > 0$.

$\sin\frac{\theta}{2} = \sqrt{\frac{1 - \cos\theta}{2}} = \sqrt{\frac{1 - (-\frac{1}{4})}{2}} = \sqrt{\frac{5}{8}}$

$= \frac{\sqrt{5}}{2\sqrt{2}} = \frac{\sqrt{10}}{4}$

$\cos\frac{\theta}{2} = \sqrt{\frac{1 + \cos\theta}{2}} = \sqrt{\frac{1 + (-\frac{1}{4})}{2}} = \sqrt{\frac{3}{8}}$

$= \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{6}}{4}$

$\tan\frac{\theta}{2} = \sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = \sqrt{\frac{1 - (-\frac{1}{4})}{1 + (-\frac{1}{4})}}$

$= \sqrt{\frac{5}{8} \div \frac{3}{8}} = \sqrt{\frac{5}{3}} = \frac{\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{15}}{3}$

42. 22.5°, or $\frac{\pi}{8}$

a. $\sin 22.5° = \sqrt{\frac{1 - \cos 45°}{2}} = \sqrt{\frac{1}{2}\left(1 - \frac{\sqrt{2}}{2}\right)}$

$= \sqrt{\frac{2 - \sqrt{2}}{4}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$

b. $\cos 22.5° = \sqrt{\frac{1 + \cos 45°}{2}} = \sqrt{\frac{1}{2}\left(1 + \frac{\sqrt{2}}{2}\right)}$

$= \sqrt{\frac{2 + \sqrt{2}}{4}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$

c. $\tan 22.5° = \sqrt{\frac{1 - \cos 45°}{1 + \cos 45°}} = \frac{1 - \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2 - \sqrt{2}}{\sqrt{2}}$

$= \frac{2\sqrt{2} - 2}{2} = \sqrt{2} - 1$

44. $\sin 37.5° = \sin(15° + 22.5°)$

$= \sin 15°\cos 22.5° + \cos 15°\sin 22.5°$

$= \frac{\sqrt{2 - \sqrt{3}}}{2}\cdot\frac{\sqrt{2 + \sqrt{2}}}{2} + \frac{\sqrt{2 + \sqrt{3}}}{2}\cdot\frac{\sqrt{2 - \sqrt{2}}}{2}$

$= \frac{\sqrt{4 - \sqrt{6} - 2\sqrt{3} + 2\sqrt{2}} + \sqrt{4 - \sqrt{6} + 2\sqrt{3} - 2\sqrt{2}}}{4}$

Note: $\sqrt{2 - \sqrt{3}}\cdot\sqrt{2 + \sqrt{2}} = \sqrt{(2 - \sqrt{3})(2 + \sqrt{2})}$

$= \sqrt{4 + 2\sqrt{2} - 2\sqrt{3} - \sqrt{6}}$

The calculation for $\sqrt{2 + \sqrt{3}}\cdot\sqrt{2 - \sqrt{2}}$ is similar.

80. Find identities for $\sin 4\theta$ in terms of $\sin x$ and $\cos x$, and for $\cos 4\theta$ in terms of $\cos\theta$.

a. $\sin 4\theta = \sin 2(2\theta)$

$= 2\sin 2\theta\cos 2\theta$

$= 2(2\sin\theta\cos\theta)(1 - 2\sin^2\theta)$

$= 4\sin\theta\cos\theta - 8\sin^3\theta\cos\theta$

Depending on how $\cos 2\theta$ is expanded, other possible answers are

$\sin 4\theta = 8\cos^3\theta\sin\theta - 4\cos\theta\sin\theta$

$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\sin^3\theta\cos\theta$

b. $\cos 4\theta = \cos 2(2\theta)$

$= 2\cos^2(2\theta) - 1$

$= 2[\cos 2\theta]^2 - 1$

$= 2[2\cos^2\theta - 1]^2 - 1$

$= 2[4\cos^4\theta - 4\cos^2\theta + 1] - 1$

$= 8\cos^4\theta - 8\cos^2\theta + 1$

89. $\cos\theta_2 = \cos\frac{\theta}{2} = \frac{8}{9}$; $\cos\theta = \frac{8}{x}$

$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$

The angles are acute, so we need the positive value.

$\frac{8}{9} = \sqrt{\frac{1+\frac{8}{x}}{2}}$

Substitute values.

$\frac{64}{81} = \frac{1+\frac{8}{x}}{2}$

Square both members.

$128 = 81\left(1+\frac{8}{x}\right)$

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

$128 = 81 + \frac{648}{x}$

$47 = \frac{648}{x}$

$x = \frac{648}{47} = 13\frac{37}{47}$

Exercise 5–4

Answers to odd-numbered problems

1. $\frac{3\pi}{4}(135°), \frac{7\pi}{4}(315°)$ **3.** $\frac{\pi}{3}(60°), \frac{5\pi}{3}(300°)$

5. $\frac{\pi}{6}(30°), \frac{7\pi}{6}(210°)$ **7.** $\frac{\pi}{3}(60°), \frac{5\pi}{3}(300°)$

9. $\frac{\pi}{2}(90°), \frac{3\pi}{2}(270°)$ **11.** $0(0°), \pi(180°)$ **13.** $0(0°), \frac{3\pi}{2}(270°)$

15. $\frac{\pi}{4}(45°), \frac{3\pi}{4}(135°), \frac{5\pi}{4}(225°), \frac{7\pi}{4}(315°)$

17. $0(0°), \pi(180°), \frac{\pi}{2}(90°)$ **19.** $0(0°), \pi(180°), \frac{\pi}{3}(60°), \frac{4\pi}{3}(240°)$ **21.** $\frac{3\pi}{2}(270°), \frac{\pi}{6}(30°), \frac{5\pi}{6}(150°)$

23. $0(0°), \pi(180°), \frac{\pi}{4}(45°), \frac{3\pi}{4}(135°), \frac{5\pi}{4}(225°), \frac{7\pi}{4}(315°)$

25. $0(0°), \pi(180°), \frac{\pi}{3}(60°), \frac{5\pi}{3}(300°)$

27. $\frac{5\pi}{6}(150°), \frac{11\pi}{6}(330°), \frac{\pi}{2}(90°), \frac{3\pi}{2}(270°)$

29. $0(0°), \pi(180°), \frac{\pi}{2}(90°), \frac{3\pi}{2}(270°)$

31. $\frac{\pi}{6}(30°), \frac{5\pi}{6}(150°), \frac{3\pi}{2}(270°)$ **33.** $\frac{3\pi}{4}(135°), \frac{7\pi}{4}(315°)$

35. $\frac{\pi}{3}(60°), \frac{5\pi}{3}(300°), \pi(180°)$

37. $\frac{\pi}{3}(60°), \frac{2\pi}{3}(120°), \frac{4\pi}{3}(240°), \frac{5\pi}{3}(300°)$

39. $\frac{\pi}{4}(45°), \frac{3\pi}{4}(135°), \frac{5\pi}{4}(225°), \frac{7\pi}{4}(315°)$

41. $0(0°), \pi(180°), \frac{\pi}{6}(30°), \frac{5\pi}{6}(150°)$

43. 0.65, 2.49, 3.42, 6.01 (37.4°, 142.6°, 195.9°, 344.1°)

45. 0.27, 2.08, 3.42, 5.22 (15.7°, 119.3°, 195.7°, 299.3°)

47. 1.26, 2.51, 3.77, 5.03 (72°, 144°, 216°, 288°)

49. 5.08(290.8°), 0.56(32.3°)

51. $\frac{\pi}{3} + 2k\pi(60° + k \cdot 360°), \frac{5\pi}{3} + 2k\pi(300° + k \cdot 360°)$

53. $\frac{5\pi}{6} + k\pi, (150° + k \cdot 180°)$

55. $\frac{5\pi}{4} + 2k\pi(225° + k \cdot 360°), \frac{7\pi}{4} + 2k\pi(315° + k \cdot 360°)$

57. $\frac{\pi}{4} + k\pi(45° + k \cdot 180°)$

59. $\frac{\pi}{6} + 2k\pi(30° + k \cdot 360°), \frac{5\pi}{6} + 2k\pi(150° + k \cdot 360°)$

61. $\frac{2\pi}{3} + 4k\pi(120° + k \cdot 720°), \frac{4\pi}{3} + 4k\pi(240° + k \cdot 720°)$

63. $\frac{\pi}{3} + \frac{2k\pi}{3}(60° + k \cdot 120°)$

65. $\frac{\pi}{6} + k\frac{\pi}{2}(30° + k \cdot 90°)$

67. $\frac{\pi}{6} + k\frac{\pi}{2}(30° + k \cdot 90°), \frac{\pi}{3} + k\frac{\pi}{2}(60° + k \cdot 90°)$

69. $\frac{\pi}{3} + k\pi(60° + k \cdot 180°), \frac{2\pi}{3} + k\pi(120° + k \cdot 180)$

71. $\frac{\pi}{12} + k\frac{\pi}{2}(15° + k \cdot 90°)$

73. $\frac{\pi}{12} + k\pi(15° + k \cdot 180°), \frac{5\pi}{12} + k\pi(75° + k \cdot 180°)$

75. $\frac{\pi}{9} + \frac{2k\pi}{3}(20° + k \cdot 120°), \frac{5\pi}{9} + \frac{2k\pi}{3}(100° + k \cdot 120°)$

77. $\frac{2\pi}{3} + 4k\pi(120° + k \cdot 720°)$

79. $\frac{7\pi}{6}(210°), \frac{11\pi}{6}(330°), \frac{\pi}{2}(90°)$

81. $0(0°), \pi(180°), \frac{2\pi}{3}(120°), \frac{4\pi}{3}(240°)$

83. $0(0°), \pi(180°), \frac{\pi}{6}(30°), \frac{5\pi}{6}(150°)$

85. $0(0°)$ **87.** $\frac{\pi}{3}(60°)$ or $\frac{5\pi}{3}(300°)$

89. $0(0°), \frac{3\pi}{2}(270°)$ **91.** $\frac{\pi}{2}(90°), \frac{3\pi}{2}(270°), \pi(180°), 0.93(53.1°)$

93. $\frac{\pi}{2}$, 1.31 **95.** 0, 0.55, 0.69, 0.72, 0.66

Solutions to trial exercise problems

4. $2 \cos \theta + 1 = 0$
$2 \cos \theta = -1$
$\cos \theta = -\frac{1}{2}$
$\theta' = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}(60°)$
$\cos \theta < 0$ in quadrants II and III, so $\theta = \pi - \theta' =$
$\pi - \frac{\pi}{3} = \frac{2\pi}{3}(180° - 60° = 120°)$ or $\theta = \pi + \theta' =$
$\pi + \frac{\pi}{3} = \frac{4\pi}{3}(180° + 60° = 240°)$.

20. $\cos^2\theta - \frac{1}{2} \cos \theta = 0$
$\cos \theta(\cos \theta - \frac{1}{2}) = 0$
Case 1: $\cos \theta = 0$
$\frac{\pi}{2}(90°), \frac{3\pi}{2}(270°)$
Case 2: $\cos \theta - \frac{1}{2} = 0$
$\cos \theta = \frac{1}{2}$
$\cos \theta > 0$ in quadrants I and IV, so $\theta = \theta' = \frac{\pi}{3}(60°)$
or $2\pi - \theta' = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}(360° - 60° = 300°)$.

27. $\sqrt{3} \tan \theta \cot \theta + \cot \theta = 0$
$\cot \theta(\sqrt{3} \tan \theta + 1) = 0$
Case 1: $\cot \theta = 0$
$\frac{\cos \theta}{\sin \theta} = 0$
$\cos \theta = 0$
$\frac{\pi}{2}(90°)$ or $\frac{3\pi}{2}(270°)$
Case 2: $\sqrt{3} \tan \theta + 1 = 0$
$\tan \theta = -\frac{\sqrt{3}}{3}$
$\theta' = \tan^{-1}\frac{\sqrt{3}}{3} = \frac{\pi}{6}(30°)$.
$\tan \theta < 0$ in quadrants II and IV, so $\theta = \pi - \theta' =$
$\pi - \frac{\pi}{6} = \frac{5\pi}{6}(180° - 30° = 150°)$ or $2\pi - \theta' =$
$2\pi - \frac{\pi}{6} = \frac{11\pi}{6}(360° - 30° = 330°)$.

33. $\tan x + \cot x = -2$
$\tan x + \frac{1}{\tan x} = -2$
$\tan^2x + 1 = -2 \tan x$
Multiply each member by $\tan x$; assume $\tan x \neq 0$.
$\tan^2x + 2 \tan x + 1 = 0$
$(\tan x + 1)^2 = 0$
$\tan x + 1 = 0$
$\tan x = -1$
$\theta' = \tan^{-1}1 = \frac{\pi}{4}(45°)$
Tangent is negative in quadrants II and IV, so $\theta = \pi - \theta' =$
$\pi - \frac{\pi}{4} = \frac{3\pi}{4}(180° - 45° = 135°)$ or $2\pi - \theta' =$
$2\pi - \frac{\pi}{4} = \frac{7\pi}{4}(360° - 45° = 315°)$.

34. $2 - \sin x - \csc x = 0$
$2 - \sin x - \frac{1}{\sin x} = 0$
$2 \sin x - \sin^2x - 1 = 0$
$-\sin^2x + 2 \sin x - 1 = 0$
$\sin^2x - 2 \sin x + 1 = 0$
$(\sin x - 1)^2 = 0$
$\sin x - 1 = 0$
$\sin x = 1$
$x = \frac{\pi}{2}(90°)$

41. $2 \tan^2x \sin x = \tan^2x$
$2 \tan^2x \sin x - \tan^2x = 0$
$\tan^2x(2 \sin x - 1) = 0$
Case 1: $\tan^2x = 0$
$\tan x = 0$
$\frac{\sin \theta}{\cos \theta} = 0$
$\sin \theta = 0$
$0(0°), \pi(180°)$
Case 2: $2 \sin x - 1 = 0$
$\sin x = \frac{1}{2}$
$\theta' = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$
$\sin \theta > 0$ in quadrants I and II.
$\theta = \theta' = \frac{\pi}{6}(30°)$ or
$\pi - \frac{\pi}{6} = \frac{5\pi}{6}(150°)$

Recall: If $ax^2 + bx + c = 0, c \neq 0$, then
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

46. $\tan^2x + 5 \tan x + 2 = 0$
$a = 1, b = 5, c = 2$:
$\tan x = \frac{-5 \pm \sqrt{5^2 - 4(2)}}{2} = \frac{-5 \pm \sqrt{17}}{2}$
Case 1:
$\tan x = \frac{-5 + \sqrt{17}}{2} \approx -0.4384$
$x = \tan^{-1}\left|\frac{-5 + \sqrt{17}}{2}\right| \approx 0.413(23.7°)$
$\tan x < 0$ in quadrants II and IV.
$x = \pi - x' \approx \pi - 0.413 \approx 2.74$
$= 2\pi - x' \approx 2\pi - 0.413 \approx 5.87$
$x = 180° - x' \approx 180° - 23.7° \approx 156.3°$
$= 360° - x' \approx 360° - 23.7° \approx 336.3°$

Case 2:

$\tan x = \dfrac{-5 - \sqrt{17}}{2} \approx -4.5616$

$x = \tan^{-1}\left|\dfrac{-5 - \sqrt{17}}{2}\right| \approx 1.355(77.6°)$

$\tan x < 0$ in quadrants II and IV.

$x = \pi - x' \approx \pi - 1.355 \approx 1.79$

$= 2\pi - x' \approx 2\pi - 1.355 \approx 4.93$

$x = 180° - x' \approx 180° - 77.6° \approx 102.4°$

$= 360° - x' \approx 360° - 77.6° \approx 282.4°$

57. $\tan x = 1$

$x = \tan^{-1} 1 = \dfrac{\pi}{4}(45°)$

Primary solutions are in quadrants I and III:

$\dfrac{\pi}{4}(45°)$ and $\dfrac{5\pi}{4}(225°)$. These differ by $\pi(180°)$, so we can write all solutions with one of them: $\dfrac{\pi}{4} + k\pi(45° + k \cdot 180°)$.

64. $\sec \dfrac{x}{2} = 1$; $\cos \dfrac{x}{2} = 1$

Primary solutions: $\dfrac{x}{2} = \cos^{-1} 1 = 0(0°)$

All solutions: $\dfrac{x}{2} = 0 + 2k\pi(0° + k \cdot 360°)$

$x = 4k\pi(k \cdot 720°)$

74. $\sin \dfrac{\theta}{3} = \dfrac{\sqrt{3}}{2}$

$\left(\dfrac{\theta}{3}\right)' = \sin^{-1}\dfrac{\sqrt{3}}{2} = \dfrac{\pi}{3}(60°)$

Primary solutions: $\dfrac{\theta}{3} = \dfrac{\pi}{3}(60°)$ or $\dfrac{2\pi}{3}(120°)$

All solutions: $\dfrac{\theta}{3} = \dfrac{\pi}{3} + 2k\pi(60° + k \cdot 360°)$ or

$\dfrac{2\pi}{3} + 2k\pi(120° + k \cdot 360°)$

$\theta = \pi + 6k\pi(180° + k \cdot 1{,}080°)$ or

$2\pi + 6k\pi(360° + k \cdot 1{,}080°)$.

81. $\sin 2\theta + \sin \theta = 0$

$2 \sin \theta \cos \theta + \sin \theta = 0$

$\sin \theta(2 \cos \theta + 1) = 0$

Case 1: $\sin \theta = 0$

$0(0°), \pi(180°)$

Case 2: $2 \cos \theta + 1 = 0$

$\cos \theta = -\frac{1}{2}$

$\dfrac{2\pi}{3}(120°)$ or $\dfrac{4\pi}{3}(240°)$

88. $\sin^2 \dfrac{\theta}{2} = \cos \theta$

$\left(\pm\sqrt{\dfrac{1 - \cos \theta}{2}}\right)^2 = \cos \theta$

$\dfrac{1 - \cos \theta}{2} = \cos \theta$

$1 - \cos \theta = 2 \cos \theta$

$1 = 3 \cos \theta$

$\frac{1}{3} = \cos \theta$

$\theta' = \cos^{-1}\frac{1}{3}$

$\theta = \cos^{-1}\frac{1}{3}$ or $2\pi - \cos^{-1}\frac{1}{3}$

$\theta = 1.23(70.5°)$ or $5.05(289.5°)$

92. If $A = 0.855$, $B = 1.052$, and $y = 0$, solve for x to the nearest 0.01.

$0 = x \cos 0.855 \cos 1.052 - x^2 \cos 0.855 \sin 1.052$

$- x^3 \sin 0.855$

$0 = 0.32538x - 0.56987x^2 - 0.75457x^3$

$x(0.75457x^2 + 0.56987x - 0.32538) = 0$

$x = 0$ or $0.75457x^2 + 0.56987x - 0.32538 = 0$

Solve the quadratic equation with the quadratic formula.

$x = 0, -1.14, 0.38$

93. If $B = 0.7$, $x = 2$, and $y = -8$, find A to the nearest 0.01.

$-8 = 2 \cos A \cos 0.7 - 4 \cos A \sin 0.7 - 8 \sin A$

$-8 = (2 \cos 0.7)\cos A - (4 \sin 0.7)\cos A - 8 \sin A$

$-8 = 1.5297 \cos A - 2.5769 \cos A - 8 \sin A$

$-8 = -1.0472 \cos A - 8 \sin A$

$8 \sin A - 8 = -1.0472 \cos A$

$\sin A - 1 = -0.1309 \cos A$

Divide each member by 8.

$\sin A = 1 - 0.1309 \cos A$

$(\sin A)^2 = (1 - 0.1309 \cos A)^2$

$\sin^2 A = 1 - 0.2618 \cos A + 0.017134 \cos^2 A$

$1 - \cos^2 A = 1 - 0.2618 \cos A + 0.017134 \cos^2 A$

$0 = 1.0171 \cos^2 A - 0.2618 \cos A$

$0 = \cos A(1.0171 \cos A - 0.2618)$

$\cos A = 0$ or $1.0171 \cos A - 0.2618 = 0$

$A = \cos^{-1} 0$ or $1.0171 \cos A = 0.2618$

$A = \dfrac{\pi}{2}$ or $\cos A = 0.25739$

$A \approx 1.310476103$

Thus, A is $\dfrac{\pi}{2}$ or 1.31

96. Find θ when $\mathcal{A} = 0.5\ \text{m}^2$. Round the answer to the nearest 0.1°.

$\sin \theta\sqrt{1.44 - \sin^2\theta} = 0.5$

$(\sin \theta\sqrt{1.44 - \sin^2\theta})^2 = 0.5^2$

$\sin^2\theta(1.44 - \sin^2\theta) = 0.25$

$1.44 \sin^2\theta - \sin^4\theta = 0.25$

$\sin^4\theta - 1.44 \sin^2\theta + 0.25 = 0$

Let $u = \sin^2\theta$.

$u^2 - 1.44u + 0.25 = 0$

$u \approx 1.2381$ or $u \approx 0.20193$

$\sin^2\theta \approx 1.2381$ or $\sin^2\theta \approx 0.20193$

$\sin \theta \approx \pm\sqrt{1.2381}$ or $\sin \theta \approx \pm\sqrt{0.20193}$

$\sin \theta \approx \pm 1.1127$ or $\sin \theta \approx \pm 0.44936$

no solution or $\theta \approx \pm 26.7°$

Because $\theta \geq 0$, we choose the positive value for θ.

$\theta \approx 26.7°$

Chapter 5 review

1.
$$\frac{\frac{\cos\theta}{\sin\theta}}{\cos\theta}$$
$$\frac{1}{\sin\theta}$$
$$\csc\theta$$

2.
$$\sec\theta\frac{\sin\theta}{\cos\theta}$$
$$\sec\theta\left(\frac{1}{\cos\theta}\right)\sin\theta$$
$$\sec\theta\sec\theta\sin\theta$$
$$\sec^2\theta\sin\theta$$

3.
$$\frac{\tan^2\theta}{\tan^2\theta}$$
$$1$$
$$\sin^4\theta\frac{1}{\sin^4\theta}$$
$$\sin^4\theta\csc^4\theta$$

4.
$$\frac{\cot^2\theta}{\tan^2\theta}$$
$$\frac{\frac{\cos^2\theta}{\sin^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}$$
$$\frac{\cos^4\theta}{\sin^4\theta}$$
$$\cot^4\theta$$

5.
$$\frac{\csc\theta\frac{\sin\theta}{\cos\theta}}{\sin\theta}$$
$$\csc\theta\frac{1}{\cos\theta}$$
$$\csc\theta\sec\theta$$

6.
$$\sin^2\theta-(1-\sin^2\theta)$$
$$\sin^2\theta-1+\sin^2\theta$$
$$2\sin^2\theta-1$$

7.
$$\frac{1}{\sin\theta}-\frac{1}{\cos\theta}$$
$$\frac{\cos\theta-\sin\theta}{\sin\theta\cos\theta}$$

8.
$$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$$
$$\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$$
$$\frac{1}{\cos\theta\sin\theta}$$

9.
$$\frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}-\frac{\cos\theta}{\sin\theta}}$$
$$\frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta-\cos^2\theta}{\cos\theta\sin\theta}}$$
$$\frac{1}{\frac{\sin^2\theta-\cos^2\theta}{\sin\theta}}$$
$$\frac{\sin\theta}{\sin^2\theta-\cos^2\theta}$$

10.
$$\frac{1-\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}+1}$$
$$\frac{\frac{\sin\theta-\cos\theta}{\sin\theta}}{\frac{1+\sin\theta}{\sin\theta}}$$
$$\frac{\sin\theta-\cos\theta}{1+\sin\theta}$$

11.
$$\frac{\frac{1}{\cos^2\theta}-1}{\frac{1}{\cos^2\theta}}$$
$$\frac{\frac{1-\cos^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$$
$$1-\cos^2\theta$$
$$\sin^2\theta$$

12.
$$\frac{1-\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\sin^2\theta}-1}$$
$$\frac{\frac{\sin^2\theta-\cos^2\theta}{\sin^2\theta}}{\frac{1-\sin^2\theta}{\sin^2\theta}}$$
$$\frac{\sin^2\theta-\cos^2\theta}{1-\sin^2\theta}$$

13.
$$\csc x-\tan x\cot x$$
$$\csc x-\tan x\left(\frac{1}{\tan x}\right)$$
$$\csc x-1$$

14.
$$\sin^2x+\sin^2x\cot^2x$$
$$\sin^2x+\sin^2x\frac{\cos^2x}{\sin^2x}$$
$$\sin^2x+\cos^2x$$
$$1$$

15.
$$\csc^2x\sec^2x(\cos^2x-\sin^2x)$$
$$\left(\frac{1}{\sin^2x}\right)\left(\frac{1}{\cos^2x}\right)(\cos^2x-\sin^2x)$$
$$\frac{1}{\sin^2x}-\frac{1}{\cos^2x}$$
$$\csc^2x-\sec^2x$$

16.
$$\frac{1}{\csc x-\cot x}$$
$$\frac{1}{\frac{1}{\sin x}-\frac{\cos x}{\sin x}}$$
$$\frac{1}{\frac{1-\cos x}{\sin x}}$$
$$\frac{\sin x}{1-\cos x}$$

17.
$$\sec x(\sin x-\cos x)$$
$$\frac{1}{\cos x}(\sin x-\cos x)$$
$$\frac{\sin x}{\cos x}-\frac{\cos x}{\cos x}$$
$$\tan x-1$$

18.
$$\frac{\csc^2x-1}{\sin^2x}$$
$$\frac{\cot^2x}{\sin^2x}$$
$$\frac{\frac{\cos^2x}{\sin^2x}}{\sin^2x}$$
$$\frac{\cos^2x}{\sin^4x}$$
$$\cos^2x\csc^4x$$

19.
$$\frac{1}{1+\csc x}+\frac{1}{1-\csc x}$$
$$\frac{1-\csc x+1+\csc x}{1-\csc^2x}$$
$$\frac{2}{1-\csc^2x}$$
$$\frac{2}{-1(\csc^2x-1)}$$
$$\frac{-2}{\cot^2x}$$
$$-2\tan^2x$$

20.
$$\sin^2x+\sin^2x\cos^2x$$
$$\sin^2x(1+\cos^2x)$$
$$(1-\cos^2x)(1+\cos^2x)$$
$$1-\cos^4x$$

21.
$$\frac{\sec^2x}{\cot^2x}$$
$$\frac{\tan^2x+1}{\cot^2x}$$
$$\frac{\tan^2x+1}{\frac{1}{\tan^2x}}$$
$$\tan^4x+\tan^2x$$

22.
$$\frac{1-\cot x}{1+\csc x}$$
$$\frac{1-\frac{\cos x}{\sin x}}{1+\frac{1}{\sin x}}$$
$$\frac{\frac{\sin x-\cos x}{\sin x}}{\frac{\sin x+1}{\sin x}}$$
$$\frac{\sin x-\cos x}{\sin x+1}$$

23.
$$\sin\frac{\pi}{6}+\cos\frac{\pi}{6}$$
$$\frac{1}{2}+\frac{\sqrt{3}}{2}$$
$$\frac{1+\sqrt{3}}{2}\neq 1$$

24.
$$\tan\frac{\pi}{4}-\sin\frac{\pi}{4}\cos\frac{\pi}{4}$$
$$1-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$
$$1-\frac{1}{2}$$
$$\frac{1}{2}\neq 0$$

25.
$$\sec\frac{\pi}{4}=\sqrt{2}$$
$$\frac{1}{\cot\frac{\pi}{4}-\csc\frac{\pi}{4}}=\frac{1}{1-\sqrt{2}}$$
$$=\frac{1+\sqrt{2}}{-1}=-1-\sqrt{2}\neq\sqrt{2}$$

26. $\dfrac{\sqrt{2}+\sqrt{6}}{4}$ **27.** $\sqrt{3}-2$

28. $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ **29.** $-2+\sqrt{3}$

30. a. $\dfrac{-15+\sqrt{77}}{24}$ **b.** $\dfrac{-15-\sqrt{77}}{5\sqrt{7}-3\sqrt{11}}$

31. a. $\dfrac{3\sqrt{5}+1}{8}$ **b.** $\dfrac{\sqrt{15}+\sqrt{3}}{1-3\sqrt{5}}$

32. a. $\dfrac{204}{8\sqrt{119}+75}$

b. $\dfrac{952-75\sqrt{119}}{-40\sqrt{119}-1{,}785}$

33. $\dfrac{\sin(\alpha+\beta)}{\cos(\alpha-\beta)}$

$\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta+\sin\alpha\sin\beta}$

$\dfrac{\dfrac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta}+\dfrac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta}}{\dfrac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}+\dfrac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}}$

$\dfrac{\cot\beta+\cot\alpha}{\cot\alpha\cot\beta+1}$

34. $\cos\left(\dfrac{3\pi}{2}+\theta\right)$

$\cos\dfrac{3\pi}{2}\cos\theta-\sin\dfrac{3\pi}{2}\sin\theta$

$(0)\cos\theta-(-1)\sin\theta$

$\sin\theta$

35. $\sin\left(\dfrac{\pi}{4}-\theta\right)$

$\sin\dfrac{\pi}{4}\cos\theta-\cos\dfrac{\pi}{4}\sin\theta$

$\dfrac{\sqrt{2}}{2}\cos\theta-\dfrac{\sqrt{2}}{2}\sin\theta$

$\dfrac{\sqrt{2}}{2}(\cos\theta-\sin\theta)$

36. $\dfrac{\cos(\alpha+\beta)}{\sin\alpha\cos\beta}$

$\dfrac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\sin\alpha\cos\beta}$

$\dfrac{\cos\alpha\cos\beta}{\sin\alpha\cos\beta}-\dfrac{\sin\alpha\sin\beta}{\sin\alpha\cos\beta}$

$\dfrac{\cos\alpha}{\sin\alpha}-\dfrac{\sin\beta}{\cos\beta}$

$\cot\alpha-\tan\beta$

37. $\cot(\theta+\pi)$

$\dfrac{1}{\tan(\theta+\pi)}$

$\dfrac{1}{\dfrac{\tan\theta+\tan\pi}{1-\tan\theta\tan\pi}}$

$\dfrac{1}{\dfrac{\tan\theta+(0)}{1-\tan\theta\,(0)}}$

$\dfrac{1}{\tan\theta}$

$\cot\theta$

38. $124°$ **39.** 10π **40.** $\dfrac{7\pi}{6}$ **41.** $12°$

42. $3\sin\theta$ **43.** $2\tan 8\theta$

44. a. $-\dfrac{5\sqrt{119}}{47}$ **b.** $\dfrac{5\sqrt{119}}{72}$ **45 a.** $-\frac{7}{25}$

b. $\frac{24}{7}$ **46. a.** $-\frac{41}{40}$ **b.** $\frac{9}{40}$

47. $\sin 2x-\cos x$

$2\sin x\cos x-\cos x$

$\cos x(2\sin x-1)$

48. $1+\cos 2x$

$1+\cos^2 x-\sin^2 x$

$1+\cos^2 x-(1-\cos^2 x)$

$1+\cos^2 x-1+\cos^2 x$

$2\cos^2 x$

49. $\dfrac{\cos 2x}{2-4\sin^2 x}$

$\dfrac{\cos^2 x-\sin^2 x}{2(1-2\sin^2 x)}$

$\dfrac{\cos^2 x-\sin^2 x}{2(\sin^2 x+\cos^2 x-2\sin^2 x)}$

$\dfrac{\cos^2 x-\sin^2 x}{2(\cos^2 x-\sin^2 x)}$

$\frac{1}{2}$

50. $\dfrac{2\cot x}{\cot^2 x-1}$

$\dfrac{\dfrac{2}{\tan x}}{\dfrac{1}{\tan^2 x}-1}$

$\dfrac{\dfrac{2}{\tan x}}{\dfrac{1-\tan^2 x}{\tan^2 x}}$

$\dfrac{2\tan x}{1-\tan^2 x}$

$\tan 2x$

51. $\sin 2x-\cos 2x$

$2\sin x\cos x-(\cos^2 x-\sin^2 x)$

$2\sin x\cos x-\cos^2 x+\sin^2 x$

$2\sin x\cos x-\cos^2 x+1-\cos^2 x$

$2\sin x\cos x-2\cos^2 x+1$

$2\cos x(\sin x-\cos x)+1$

52. $\sqrt{2}-1$ **53.** $\dfrac{\sqrt{2+\sqrt{3}}}{2}$

54. $\dfrac{\sqrt{2+\sqrt{3}}}{2}$ **55.** $\dfrac{\sqrt{2-\sqrt{2}}}{2}$

56. a. $\dfrac{\sqrt{26}}{26}$ **b.** $\dfrac{1}{5}$ **57. a.** $-\sqrt{\dfrac{6}{3-\sqrt{5}}}$

b. $\dfrac{-3+\sqrt{5}}{2}$ **58.** $\dfrac{5}{6}$

59. $\cot\dfrac{\theta}{2}$

$\dfrac{1}{\tan\dfrac{\theta}{2}}$

$\dfrac{1}{\dfrac{\sin\theta}{1+\cos\theta}}$

$\dfrac{1+\cos\theta}{\sin\theta}$

60. $\sec^2\dfrac{\theta}{2}-\tan^2\dfrac{\theta}{2}$

$\left(1+\tan^2\dfrac{\theta}{2}\right)-\tan^2\dfrac{\theta}{2}$

$1+\tan^2\dfrac{\theta}{2}-\tan^2\dfrac{\theta}{2}$

1

61. $\tan\dfrac{\theta}{2}\csc^2\dfrac{\theta}{2}$

$\dfrac{\sin\theta}{1+\cos\theta}\cdot\dfrac{2}{1-\cos\theta}$

$\dfrac{2\sin\theta}{1-\cos^2\theta}$

$\dfrac{2\sin\theta}{\sin^2\theta}$

$\dfrac{2}{\sin\theta}$

62. $\dfrac{\pi}{6}$ **63.** $\dfrac{\pi}{3}$

64. $\dfrac{\pi}{3},\dfrac{\pi}{2}$ **65.** $\dfrac{\pi}{6},\dfrac{\pi}{3}$

66. 45°, 90°, 225°, 270° **67.** 60°, 120°, 240°, 300° **68.** 0°, 120°, 240° **69.** 30°, 150°, 270° **70.** 30°, 90°, 150°, 270° **71.** 120°, 180°, 240° **72.** $k\pi$

73. $1.31 + k\pi, -0.67 + k\pi$
74. $3.52 + 2k\pi, 5.90 + 2k\pi$
75. $\frac{\pi}{3} + 2k\pi, \pi + 2k\pi, \frac{5\pi}{3} + 2k\pi$
76. $\frac{\pi}{24}, \frac{5\pi}{24}, \frac{13\pi}{24}, \frac{17\pi}{24}, \frac{25\pi}{24}, \frac{29\pi}{24}, \frac{37\pi}{24}, \frac{41\pi}{24}$ **77.** $\frac{\pi}{3}$ **78.** $\frac{\pi}{2}$ **79.** π
80. 12°, 60°, 84°, 132°, 156°, 204°, 228°, 276°, 300°, 348° **81.** 240° **82.** 7.5°, 37.5°, 67.5°, 97.5°, 127.5°, 157.5°, 187.5°, 217.5°, 247.5°, 277.5°, 307.5°, 337.5°, 22.5°, 52.5°, 82.5°, 112.5°, 142.5°, 172.5°, 202.5°, 232.5°, 262.5°, 292.5°, 322.5°, 352.5° **83.** 90°, 210°, 270°, 330°
84. $\frac{\pi}{3} + 2k\pi, \pi + 2k\pi, \frac{5\pi}{3} + 2k\pi$
85. $0.13 + k \cdot \frac{\pi}{4}, 1.22 + k \cdot \frac{2\pi}{3}, 1.92 + k \cdot \frac{2\pi}{3}$ **86.** $\frac{\pi}{4} + k\pi, 1.94 + k\pi, 2.78 + k\pi$

Chapter 5 test

1. $\csc^2 x \sin x \cos x$

$\frac{1}{\sin^2 x} \sin x \cos x$

$\frac{\cos x}{\sin x}$

$\cot x$

2. $\dfrac{\frac{1}{\sin x} - \frac{1}{\cos x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$

$\dfrac{\frac{\cos x - \sin x}{\sin x \cos x}}{\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}}$

$\frac{\cos x - \sin x}{\sin^2 x + \cos^2 x}$

$\cos x - \sin x$

3. $\cot x - 2 \tan x$

$\cot 45° - 2 \tan 45°$

$1 - 2(1)$

-1

$-1 \neq 1$

4. $\frac{8\sqrt{3} - 15}{34}$ **5.** $-\frac{240}{289}$

6. 3 **7.** $\frac{2}{\sqrt{2 + \sqrt{2}}}$

8. a. $\frac{1 + \cot\theta}{\csc\theta}$

$\frac{1}{\csc\theta} + \frac{\cot\theta}{\csc\theta}$

$\dfrac{1}{\frac{1}{\sin\theta}} + \dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$

$\sin\theta + \cos\theta$

b. $\frac{\cos^2 x - 1}{\sin^2 x}$

$\frac{-(1 - \cos^2 x)}{\sin^2 x}$

$\frac{-\sin^2 x}{\sin^2 x}$

-1

c. $\cos\left(\theta - \frac{3\pi}{2}\right)$

$\cos\theta \cos\frac{3\pi}{2} + \sin\theta \sin\frac{3\pi}{2}$

$\cos\theta(0) + \sin\theta(-1)$

$-\sin\theta$

d. $-1 + 2\cos x(\cos x - \sin x)$

$-(\sin^2 x + \cos^2 x) + 2\cos^2 x - 2\sin x \cos x$

$\cos^2 x - \sin^2 x - 2\sin x \cos x$

$\cos 2x - \sin 2x$

9. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
10. 30°, 120°, 210°, 240°
11. $0.17 + 2\pi k, 2.97 + 2\pi k, \frac{3\pi}{2} + 2\pi k$
12. 18°, 54°, 90°, 126°, 162°, 198°, 234°, 270°, 306°, 342° **13.** $\pi + 8k\pi$, $7\pi + 8k\pi, 3\pi + 8k\pi, 5\pi + 8k\pi$ **14.** $\pm m$

Chapter 6

Exercise 6-1

Answers to odd-numbered problems

1. $C = 96°, b \approx 16.4, c \approx 21.7$
3. $A = 34.4, b \approx 0.52, c \approx 1.64$
5. $C = 33°, c \approx 51.2, a \approx 68.7$
7. $B = 20.6°, b \approx 0.172, c \approx 0.489$
9. $C = 35°, a \approx 8.58, b \approx 6.16$
11. $A \approx 45.6°, C \approx 85.4°, c \approx 17.4$
13. $B \approx 18.0°, C \approx 30.0°, b \approx 1.77$
15. $C \approx 47.4°, A \approx 88.9°, a \approx 133.9$
$C \approx 132.6°, A \approx 3.7°, a \approx 8.7$
17. no solution
19. $B \approx 85.0°, A \approx 52.7°, a \approx 5.07$
$B \approx 95.01°, A \approx 42.7°, a \approx 4.32$
21. $C \approx 26.23°, A \approx 108.77°, a \approx 10.71$
23. 32.3 miles **25.** 843,400 miles
27. 15.8 knots **29.** 25 miles
31. By the definitions of section 2-3, $\tan A = \frac{y}{x}$ in each figure. By the trigonometric ratios (for a right triangle) it can be seen in each case that $\tan C = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{b - x}$. (Note that x is negative in the right figure, so that $b - x$ is larger than b itself.) Also, as noted in the problem, $y = h$. Putting these values in the expression $\frac{b \cdot \tan A \cdot \tan C}{\tan A + \tan C}$ we obtain:

$$\frac{b \cdot \frac{y}{x} \cdot \frac{y}{b - x}}{\frac{y}{x} + \frac{y}{b - x}} = \frac{\frac{by^2}{x(b - x)}}{\frac{yb}{x(b - x)}} = y = h$$

33. Let (x,y) be the point at B. It is on the terminal side of angle A. Then $\cos A = \frac{x}{r}$, where r is the length of AB. But then $r = c$, so $\cos A = \frac{x}{c}$, and so $c \cos A = x$. Using right triangles we see that in each figure $\cos C = \frac{b - x}{a}$, so that $a \cos C = b - x$. Note that when A is obtuse (the right-hand figure) x is negative, so $b - x$ is the length of $|b| + |x|$. Proceeding with the information above:

$$\begin{array}{r} c\cos A = x \\ a\cos C = b - x \\ \hline a\cos C + c\cos A = x + (b - x) \\ a\cos C + c\cos A = b \end{array}$$

Thus, (2) is true.

(1) can be shown to be true by putting angle B in standard position with angle C on the x-axis and proceeding in the same manner. (2) is done with angle B in standard position and angle A on the x-axis. Actually, this is not really necessary since the labeling in a triangle is arbitrary, and thus, for example, we could obtain (1) by changing the label B to A, C to B, and A to C, and labeling the sides appropriately.

35. Consider any triangle ABC; place it as shown in the figure for problem 33, so angle A is in standard position. The figure covers the cases where A is acute, right, or obtuse. Then it can be seen that if h is the height of the triangle, then $h = x$. We know that

$\sin A = \dfrac{x}{c} = \dfrac{h}{c}$, so $h = c \sin A$.

Then the area is $\frac{1}{2}bh = \frac{1}{2}b(c \sin A) = \frac{1}{2}bc \sin A$.

37. a. It can be seen that the sum of the area of the four triangles shown in the figure is $\frac{1}{2}ab \sin A + \frac{1}{2}cd \sin C + \frac{1}{2}ad \sin D + \frac{1}{2}bc \sin B$. This total is twice as large as the total area of the four-sided figure, so the area of the four-sided figure is $\frac{1}{2}(\frac{1}{2}ab \sin A + \frac{1}{2}cd \sin C + \frac{1}{2}ad \sin D + \frac{1}{2}bc \sin B)$ or $\frac{1}{4}(ab \sin A + ad \sin D + bc \sin B + cd \sin C)$.

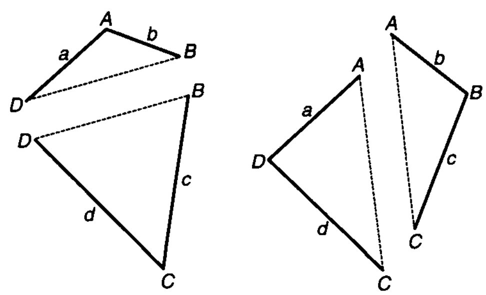
b. The difference between the Egyptian formula $\frac{1}{4}(ab + ad + bc + cd)$ and the correct formula $\frac{1}{4}(ab \sin A + ad \sin D + bc \sin B + cd \sin C)$ is the factors $\sin A$, $\sin B$, $\sin C$, and $\sin D$. The value of the sine of each angle is between 0 and 1. Thus,

$$ab \geq ab \sin A$$
$$ad \geq ad \sin D$$
$$bc \geq bc \sin B$$
$$cd \geq cd \sin C$$

$ab + ad + bc + cd \geq ab \sin A + ad \sin D + bc \sin B + cd \sin C$,

$\frac{1}{4}(ab + ad + bc + cd) \geq \frac{1}{4}(ab \sin A + ad \sin D + bc \sin B + cd \sin C)$

If the figure is a rectangle $A = B = C = D = 90°$, and $\sin A = \sin B = \sin C = \sin D = 1$, so both expressions give the same value.



Solutions to trial exercise problems

7. $a = 0.452, A = 67.6°, C = 91.8°$

$B = 180° - 67.6° - 91.8° = 20.6°$

$\dfrac{\sin 67.6°}{0.452} = \dfrac{\sin 20.6°}{b} = \dfrac{\sin 91.8°}{c}$

$\dfrac{\sin 67.6°}{0.452} = \dfrac{\sin 20.6°}{b}$

$b = \dfrac{0.452 \sin 20.6°}{\sin 67.6°}$

$b \approx 0.172$

$\dfrac{\sin 67.6°}{0.452} = \dfrac{\sin 91.8°}{c}$

$c = \dfrac{0.452 \sin 91.8°}{\sin 67.6°}$

$c \approx 0.489$

13. $a = 4.25, c = 2.86, A = 132°$

$\dfrac{\sin 132°}{4.25} = \dfrac{\sin B}{b} = \dfrac{\sin C}{2.86}$

$\dfrac{\sin 132°}{4.25} = \dfrac{\sin C}{2.86}$ so $\sin C = \dfrac{2.86 \sin 132°}{4.25}$,

$C' = \sin^{-1}\dfrac{2.86 \sin 132°}{4.25} \approx 30.01°$

$C \approx 30.01°$ or $180° - 30.01° \approx 149.99°$

Case 1: $C \approx 30.01°$

$B \approx 180° - 132° - 30.01° \approx 17.99°$

$\dfrac{\sin 132°}{4.25} \approx \dfrac{\sin 17.99°}{b}$

$b \approx 1.77$

Thus, the solution is

$B \approx 18.0°, C \approx 30.0°, b \approx 1.77$.

Case 2: $C \approx 149.99°$

$B = 180° - 132° - 149.99° \approx -101.99$

(No solution.)

15. $b = 92.5, c = 98.6, B = 43.7°$

$\dfrac{\sin A}{a} = \dfrac{\sin 43.7°}{92.5} = \dfrac{\sin C}{98.6}$

$\dfrac{\sin 43.7°}{92.5} = \dfrac{\sin C}{98.6}$, $\sin C = \dfrac{98.6 \sin 43.7°}{92.5}$

$C' = \sin^{-1}\dfrac{98.6 \sin 43.7°}{92.5} \approx 47.429°$

$C \approx 47.429°$ or $180° - 47.429° \approx 132.571°$

Case 1: $C \approx 47.429°$

$A = 180° - 43.7° - 47.429° \approx 88.871$

$\dfrac{\sin 88.871°}{a} = \dfrac{\sin 43.7°}{92.5}$

$a \approx 133.86$

Solution 1: $C \approx 47.4°$
$A \approx 88.9°$
$a \approx 133.9$

Case 2: $C \approx 132.571°$

$A = 180° - 43.7° - 132.571° \approx 3.729°$

$\dfrac{\sin 3.729°}{a} = \dfrac{\sin 43.7°}{92.5}$

$a \approx 133.60$

Solution 2: $C \approx 132.6°$
$A \approx 3.7°$
$a \approx 8.7$

27. Angle $A = 90° - 58° = 32°$.

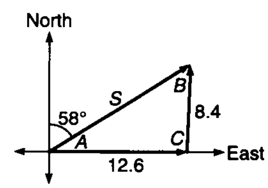
$\dfrac{\sin 32°}{8.4} = \dfrac{\sin B}{12.6}$, $\sin B = \dfrac{12.6 \sin 32°}{8.4}$,

so $B' \approx 52.64°$. Thus,

$C \approx 180° - 32° - 52.64° \approx 95.36°$.

$\dfrac{\sin 32°}{8.4} = \dfrac{\sin 95.36°}{S}$; $S \approx 15.78$.

Thus, $S \approx 15.8$ knots.



Exercise 6–2

Answers to odd-numbered problems

1. $5\sqrt{5}$ **3.** $\sqrt{226}$ **5.** $9\sqrt{2}$

7. $c = 4.0, A = 30.7°, B = 109.9°$

9. $a = 77.2, C = 14.9°, B = 41.1°$

11. $b = 38.3, C = 25.9°, A = 53.8°$

13. $C = 109.0°, B = 31.6°, A = 39.4°$

15. $C = 105.3°, A = 24.4°, B = 50.3°$

17. $c = 18.1, A = 28.3°, B = 12.3°$

19. $a = 28.1, C = 40.5°, B = 115.0°$

21. $b = 41.8, C = 26.3°, A = 41.7°$

23. $C = 110.9°, B = 30.3°, A = 38.3°$

25. $c = 0.28, A = 1.12°, B = 177.38°$

27. 326.9 feet **29.** 82.4° **31.** 125.5°

33. 102.9° **35.** 40 miles **37.** 75.0°

39. Yes, the law of cosines can be used because cos 90° is 0:
$c^2 = a^2 + b^2 - 2ab\cos C$
$c^2 = 21.3^2 + 40^2 - 2(21.3)(40)\cos 90°$
$c^2 = 21.3^2 + 40^2 - 2(21.3)(40)(0)$
$c^2 = 21.3^2 + 40^2$
$c \approx 45.3$
Since cos 90° = 0, the law of cosines is the same as the Pythagorean theorem when the angle used is 90°.

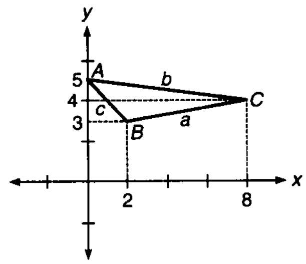
Solutions to trial exercise problems

3. $(-5,-2)$, $(10,-1)$
The distance formula is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. To use it, we must establish the values of x_1, x_2, y_1, and y_2. To find x_1 and y_1, we designate either point as the first point; we will use $(-5,-2)$. Therefore, $x_1 = -5$ and $y_1 = -2$. We now find x_2 and y_2. We use the other point for this: $x_2 = 10$ and $y_2 = -1$. We then fill these values into the formula.
$d = \sqrt{[10 - (-5)]^2 + [(-1) - (-2)]^2}$
$= \sqrt{(10 + 5)^2 + (-1 + 2)^2}$
$= \sqrt{(15)^2 + (1)^2}$
$= \sqrt{226}$

9. $b = 61.3$, $c = 23.9$, $A = 124.0°$
$a^2 = b^2 + c^2 - 2bc\cos A$
$a^2 = 61.3^2 + 23.9^2 - 2(61.3)(23.9)\cos 124°$
$a = \sqrt{61.3^2 + 23.9^2 - 2(61.3)(23.9)\cos 124°} \approx 77.249$
Ⓐ 61.3 $\boxed{x^2}$ $\boxed{+}$ 23.9 $\boxed{x^2}$ $\boxed{-}$ 2 $\boxed{\times}$ 61.3 $\boxed{\times}$ 23.9 $\boxed{\times}$ 124 $\boxed{\text{COS}}$ $\boxed{=}$ $\boxed{\sqrt{x}}$
Ⓟ 61.3 $\boxed{x^2}$ 23.9 $\boxed{x^2}$ $\boxed{+}$ 2 $\boxed{\text{ENTER}}$ 61.3 $\boxed{\times}$ 23.9 $\boxed{\times}$ 124 $\boxed{\text{COS}}$ $\boxed{\times}$ $\boxed{-}$ $\boxed{\sqrt{x}}$
$\boxed{\text{TI-81}}$ $\boxed{\sqrt{\ }}$ $\boxed{(}$ 61.3 $\boxed{x^2}$ $\boxed{+}$ 23.9 $\boxed{x^2}$ $\boxed{-}$ 2 $\boxed{\times}$ 61.3 $\boxed{\times}$ 23.9 $\boxed{\text{COS}}$ 124 $\boxed{)}$ $\boxed{\text{ENTER}}$
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
$\frac{\sin 124°}{77.249} = \frac{\sin B}{61.3} = \frac{\sin C}{23.9}$
Find angle C first; it is the smallest and therefore acute.
$\sin C \approx \frac{23.9 \sin 124°}{77.249}$; $C \approx 14.9°$
Ⓐ 23.9 $\boxed{\times}$ 124 $\boxed{\sin}$ $\boxed{\div}$ 77.249 $\boxed{=}$ $\boxed{\sin^{-1}}$
Ⓟ 23.9 $\boxed{\text{ENTER}}$ 124 $\boxed{\sin}$ $\boxed{\times}$ 77.249 $\boxed{\div}$ $\boxed{\sin^{-1}}$
$\boxed{\text{TI-81}}$ $\boxed{\text{SIN}^{-1}}$ $\boxed{(}$ 23.9 $\boxed{\text{SIN}}$ 124 $\boxed{\div}$ 77.249 $\boxed{)}$ $\boxed{\text{ENTER}}$
$B \approx 180° - 14.9° - 124° \approx 41.1°$
Thus, $a \approx 77.2$, $B \approx 41.1°$, $C \approx 14.9°$.

13. $a = 23.5$, $b = 19.4$, $c = 35.0$
$c^2 = a^2 + b^2 - 2ab\cos C$, so
$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{23.5^2 + 19.4^2 - 35^2}{2(23.5)(19.4)}$,
so $C \approx 108.97° \approx 109.0°$
Ⓐ 23.5 $\boxed{x^2}$ $\boxed{+}$ 19.4 $\boxed{x^2}$ $\boxed{-}$ 35 $\boxed{x^2}$ $\boxed{=}$ $\boxed{\div}$ 2 $\boxed{\div}$ 23.5 $\boxed{\div}$ 19.4 $\boxed{=}$ $\boxed{\cos^{-1}}$
Ⓟ 23.5 $\boxed{x^2}$ 19.4 $\boxed{x^2}$ $\boxed{+}$ 35 $\boxed{x^2}$ $\boxed{-}$ 2 $\boxed{\div}$ 23.5 $\boxed{\div}$ 19.4 $\boxed{\div}$ $\boxed{\cos^{-1}}$
$\boxed{\text{TI-81}}$ $\boxed{\text{COS}^{-1}}$ $\boxed{(}$ $\boxed{(}$ 23.5 $\boxed{x^2}$ $\boxed{+}$ 19.4 $\boxed{x^2}$ $\boxed{-}$ 35 $\boxed{x^2}$ $\boxed{)}$ $\boxed{\div}$ $\boxed{(}$ 2 $\boxed{\times}$ 23.5 $\boxed{\times}$ 19.4 $\boxed{)}$ $\boxed{)}$ $\boxed{\text{ENTER}}$
Since C is the largest angle in the triangle, we know A and B are acute.
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
$\frac{\sin A}{23.5} = \frac{\sin B}{19.4} = \frac{\sin 108.97°}{35}$
$\sin A = \frac{\sin 108.97°}{35}(23.5)$; $A \approx 39.4°$
$B \approx 180° - 39.4° - 109.0° \approx 31.6°$

31. $A(0,5)$, $B(2,3)$, $C(8,4)$



To find the largest angle, we must know which side of the triangle is longest. We find the three lengths.
$c = \sqrt{(2 - 0)^2 + (3 - 5)^2}$
$= \sqrt{8} = 2\sqrt{2}$
$b = \sqrt{(8 - 0)^2 + (4 - 5)^2}$
$= \sqrt{65}$
$a = \sqrt{(8 - 2)^2 + (4 - 3)^2}$
$= \sqrt{37}$
We now see that side b is the longest side; therefore, we need to find the measure of angle B:
$b^2 = a^2 + c^2 - 2ac\cos B$
$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
$= \frac{(\sqrt{37})^2 + (2\sqrt{2})^2 - (\sqrt{65})^2}{2(\sqrt{37})(2\sqrt{2})}$
$= \frac{37 + 8 - 65}{4(\sqrt{37})(\sqrt{2})} = -0.581238194$
$B = 125.5376778°$
Ⓐ 37 $\boxed{+}$ 8 $\boxed{-}$ 65 $\boxed{=}$ $\boxed{\div}$ 4 $\boxed{\div}$ 37 $\boxed{\sqrt{x}}$ $\boxed{\div}$ 2 $\boxed{\sqrt{x}}$ $\boxed{=}$ $\boxed{\text{INV}}$ $\boxed{\text{COS}}$
Ⓟ 37 $\boxed{\text{ENTER}}$ 8 $\boxed{+}$ 65 $\boxed{-}$ 4 $\boxed{\div}$ 37 $\boxed{\sqrt{x}}$ $\boxed{\div}$ 2 $\boxed{\sqrt{x}}$ $\boxed{\div}$ $\boxed{\cos^{-1}}$
$\boxed{\text{TI-81}}$ $\boxed{\text{COS}^{-1}}$ $\boxed{(}$ $\boxed{(}$ 37 $\boxed{+}$ 8 $\boxed{-}$ 65 $\boxed{)}$ $\boxed{\div}$ $\boxed{(}$ 4 $\boxed{\sqrt{\ }}$ 37 $\boxed{\sqrt{\ }}$ 2 $\boxed{)}$ $\boxed{)}$ $\boxed{\text{ENTER}}$
Thus, to the nearest 0.1°, the measure of B is 125.5°.

Exercise 6–3

Answers to odd-numbered problems

1. $(20\sqrt{3},20)$ **3.** $(-53.4,84.5)$

5. $(-9.4,-3.4)$ **7.** $\left(12\frac{1}{2},-\frac{25\sqrt{3}}{2}\right)$

9. $(3\sqrt{2},-3\sqrt{2})$ **11.** $(-0.8,7.8)$

13. $(5,53.1°)$ **15.** $(6.0,120.0°)$

17. $(2.6,-49.1°)$ **19.** $(11.2,-63.4°)$

21. $(7.6,153.4°)$ **23.** $(3.16,-116.6°)$

25. $(-1,20)$ **27.** $(8\sqrt{2},0)$

29. $(45.2,31.2°)$ **31.** $(36.5,-122.1°)$

33. $(9.0,-54.2°)$ **35.** $(47.1,139.7°)$

37. $(6.3,89.3°)$ **39.** $(20.3,83.9°)$

41. $(12.9,21.0°)$ **43.** The aircraft is moving west at 136 knots and north at 63 knots. **45.** The aircraft is flying east at 193 knots and north at 52 knots.

47. -228 knots = east-west component
395 knots = north-south component

49. a. 24 nm **b.** 38 nm **51.** No; the force vector is $(2{,}250,33°)$. The horizontal component f is $2{,}250 \cos 33° \approx 1{,}887$ pounds. This is not enough to move the sled.

53.

Force V	Horizontal component V_x	Vertical component V_y
(1,000,15°)	966	259
(2,000,15°)	1,932	518
(1,000,30°)	866	500

Part (a).
Part (b); yes, the components double in value.
Part (c); no, the components do not double.

55. The plane is 40 miles from Orlando, in a direction 59° south of east.

57. $(15.4,-76.8°)$ **59.** $(270.7,87.5°)$

61. The ship is about 63 knots from its starting position, at an angle of 40° south of west. **63.** The plane's ground speed is 99 knots, in the direction 47° north of west.

65. The ship is traveling at 26.5 knots in a direction 25.7° north of east.

67. Magnitude is 266 volts at 341°.

69. $(87,98°)$ The heading of the aircraft is 8° west of north, and its airspeed is 87 knots. **71.** $(17.7,76.3°)$ The ships heading is 76.3°, and its speed is 17.7 knots. **73.** $(320,130°)$ The tension in the second cable is 320 pounds, and it makes an angle θ of 50° (180° − 130°) with the horizontal.

75. Let $A = (a_1,a_2)$, $B = (b_1,b_2)$, and $C = (c_1,c_2)$ be three vectors in rectangular form. Then,

$(A + B) + C$

$= [(a_1,a_2) + (b_1,b_2)] + (c_1,c_2)$
The parentheses indicate we add A and B first.

$= (a_1 + b_1,a_2 + b_2) + (c_1,c_2)$
Two vectors; one is $A + B$, and the other is C.

$= ((a_1 + b_1) + c_1, (a_2 + b_2) + c_2)$
One vector: $(A + B) + C$.

$= (a_1 + (b_1 + c_1), a_2 + (b_2 + c_2))$
We can rearrange because real numbers are associative, and the components are real numbers.

$= A + (B + C)$

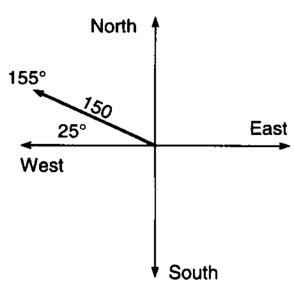
Solutions to trial exercise problems

3. $(100.0,122.3°) = (100 \cos 122.3°, 100 \sin 122.3°)$
$\approx (-53.4,84.5)$

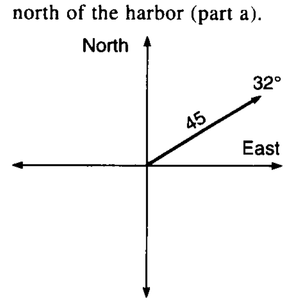
23. $(-\sqrt{2},-\sqrt{8})$ $|A| = \sqrt{(-\sqrt{2})^2 + (-\sqrt{8})} \approx \sqrt{10} \approx 3.16$,
$\theta' \approx 63.43°$; $\theta \approx 63.43° - 180° \approx -116.57°$ $(3.16,-116.6°)$

39. $(15.3,311°) = (15.3 \cos 311°,15.3 \sin 311°) = (10.038,-11.547)$
$(20.9,117°) = (20.9 \cos 117°,20.9 \sin 117°) = (-9.488,18.622)$
$(13.2,83°) = (13.2 \cos 83°,13.2 \sin 83°) = (1.609,13.102)$
$= (2.158,20.177) = (20.3,83.9°)$

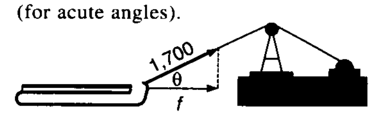
43. $V = (150,155°)$; $V_x = 150 \cos 155° \approx -136$ (136 knots due west); $V_y = 150 \sin 155° \approx 63$ (63 knots due north). The aircraft is moving west at 136 knots and north at 63 knots.



49. 18 knots (18 nautical miles per hour) × 2.5 hours = 45 nm (nautical miles); $V = (45,32°)$; $V_x = 45 \cos 32° \approx 38$ nm; distance east of the harbor (part b); $V_y = 45 \sin 32° \approx 24$ nm; distance north of the harbor (part a).



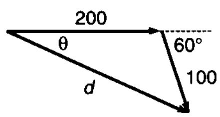
52. $f = 1{,}700 \cos \theta$. We require $f \geq 1{,}200$, so $1{,}200 \geq 1{,}700 \cos \theta$, or $\frac{12}{17} \geq \cos \theta$. $\cos^{-1}\frac{12}{17} \approx 45.1°$. Thus, $\theta \leq 45.1°$ will move the sled. Note that $\theta \leq 45.1°$ is correct, and not $\theta \geq 45.1°$. This can be seen in the figure. If θ increases, f clearly decreases. Mathematically, the value of $\cos \theta$ increases as θ decreases (for acute angles).



54. At 200 mph, the first leg of the trip is 200 miles. The second is 200 mph × 0.5 hour = 100 miles. To find d and θ, we add the vectors $(200,0°)$ and $(100,-60°)$.

$(200,0°) = (200 \cos 0°, 200 \sin 0°) = (200,0)$
$(100,-60°) = (200 \cos(-60°), 200 \sin(-60°)) \approx (50,-86.60)$
$(250,-86.60) \approx (265,341°)$

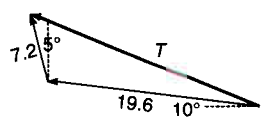
Thus, $d \approx 265$ miles, and $\theta \approx 341°$. The aircraft is 265 miles from Minneapolis. $360° - 341° = 19°$, so the aircraft is in a direction 19° south of east, relative to the city.



59. $(199,19.0°) = (199 \cos 19°, 199 \sin 19°) \approx (188.15,64.79)$
$(175,131°) = (175 \cos 131°, 175 \sin 131°) \approx (-114.81,132.07)$
$(96,130°) = (96 \cos 130°, 96 \sin 130°) \approx (-61.71,73.54)$
$(11.64,270.40) \approx (270.7,87.5°)$

64. $(19.6,170°) = (19.6 \cos 170°, 19.6 \sin 170°) \approx (-19.3,3.4)$
$(7.2,95°) = (7.2 \cos 95°, 7.2 \sin 95°) \approx (-0.63,7.17)$
$(-19.93,10.58) \approx (22.6,152.0°)$

Thus, its true course is $180° - 152° = 28°$ north of west, at a speed of 22.6 knots.



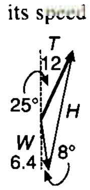
66. $(122,30°) = (122 \cos 30°, 122 \sin 30°) \approx (105.66,61.00)$
$(86,21°) = (86 \cos 21°, 86 \sin 21°) \approx (80.29,30.82)$
$(185.94,91.82) \approx (207,26°)$

Magnitude is 207 volts, phase angle is 26°.

71. We let W represent the water current vector.

$H + W = T$
$H = T - W$
$= (12,65°) - (6.4,-82°)$
$= (12,65°) + (6.4,-82° + 180°)$
$= (12,65°) + (6.4,98°)$
$= (5.07,10.88) + (-0.89,6.34)$
$= (4.18,17.21) \approx (17.7,76.3°)$

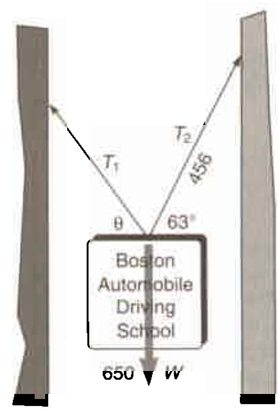
Thus, the ship's heading is 76.3°, and its speed is 17.7 knots.



73. The sign is stationary, so the forces acting on it are balanced (they add to zero).

$T_1 + T_2 + W = 0$
$T_1 = -T_2 - W$
$= -(456,63°) - (650,270°)$
$= (456,63° + 180°) + (650,270° - 180°)$
To negate a vector, add or subtract 180° from its direction angle.
$= (456,243°) + (650,90°)$
$\approx (-207.02,-406.3) + (0,650)$
Convert to rectangular form.
$\approx (-207.02,243.7)$
Convert back to polar form.
$\approx (320,130°)$

Thus, the tension in the second cable is 320 pounds, and it makes an angle θ of 50° (180° − 130°) with the horizontal.



74. Let $A = (a_1,a_2)$ and $B = (b_1,b_2)$ be two vectors in rectangular form.

Then $A + B = (a_1,a_2) + (b_1,b_2)$
$= (a_1 + b_1, a_2 + b_2)$
Definition of vector addition.
$= (b_1 + a_1, b_2 + a_2)$
a_1, a_2, b_1, b_2 are real numbers, so their indicated sum commutes.
$= B + A$
Definition of vector addition.

76. The following is for the TI-81: PRGM EDIT 2 Choose a free location to enter the program. Say 2, by way of example.

A D D V C T R S — Enter these characters as the name of the program.

```
:0→A
:0→B
:Lbl 1
:Input R
:If R=0
:Goto 2
:Input θ
:P▸R(R,θ)
:A+X→A
:B+Y→B
:Goto 1
:Lbl 2
:A→X
:B→Y
:R▸P(X,Y)
:Disp "X,Y"
:Disp X
:Disp Y
:Disp "R,θ"
:Disp R
:Disp θ
```

When running the program one inputs R, then θ, for each vector in polar form. When all vectors have been entered, enter zero (0) for R. The program converts each vector into rectangular form as it is entered, and accumulates the value (x,y) in variables A and B. When all vectors are entered, the accumulated values in A and B are converted to polar form.

Chapter 6 review

1. $C = 121.8°, b = 2.6, c = 12.1$
2. $A = 151°, a = 4.3, c = 0.8$
3. $A = 49°, C = 52°, c = 10.4$
4. $A = 76.5°, B = 70.8°, b = 12.2$; or $A = 103.5°, B = 43.8°, b = 9.0$
5. 48.2 miles **6.** $c = 3.8, A = 31.9°, B = 118.7°$ **7.** $a = 63.9, C = 18.2°, B = 69.7°$ **8.** $b = 40.2, A = 29.5°, C = 38.5°$ **9.** $A = 106.3°, B = 23.1°, C = 50.5°$ **10.** $C = 135.5°, A = 12.2°, B = 32.3°$ **11.** $c = 2\sqrt{10}, b = 7\sqrt{2}, a = \sqrt{82}, B = 77.9°, C = 38.7°, A = 63.4°$
12. 29.2 km **13.** (23.8,13.2)
14. (36.3,57.3°) **15.** $V_x = 370$ knots, $V_y = 256$ knots **16.** $V_x = 249$ lb, $V_y = 60$ lb **17.** (47.6,20.6°)
18. (10.8,89.5°) **19.** (7.2,69.9°)
20. (8.1,187.6°) **21.** 205 lb, 273°
22. 55 knots, 41° south of east
23. 183 knots, 62° north of west
24. 247 knots, 55° north of west

Chapter 6 test

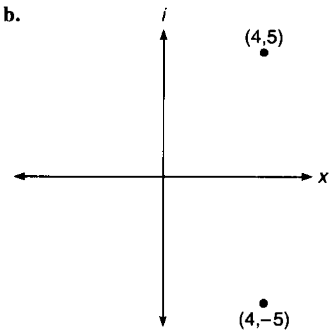
1. $B = 84.4°, a = 5.3, c = 22.5$
2. $B = 56.3°, A = 61.6°, a = 23.9$
3. $b = 32.8, A = 50.9°, C = 29.1°$
4. $C = 89.1°, A = 39.6°, B = 51.3°$
5. 59 yards **6.** 53.1° **7.** $(\sqrt{3},1)$
8. (6.4,51.3°) **9.** (8.5,84.9°)
10. 584 lb, 65°

Chapter 7

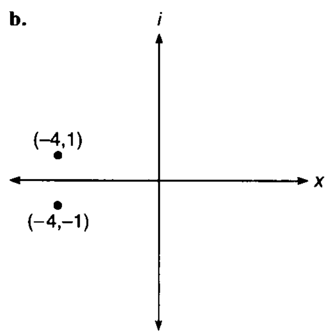
Exercise 7-1

Answers to odd-numbered problems

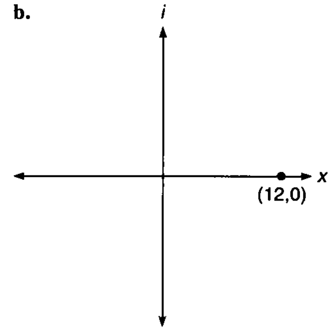
1. real, 4; imaginary, −5
3. real, −4; imaginary, 1
5. real, 12; imaginary, 0
7. real, −10; imaginary, 2
9. **a.** $4 + 5i$
b.



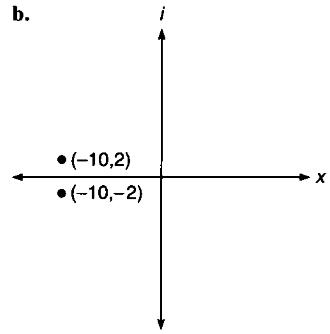
11. **a.** $-4 - i$
b.



13. **a.** 12
b.

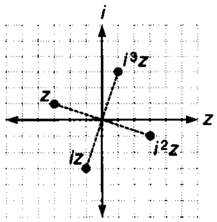


15. **a.** $-10 - 2i$
b.

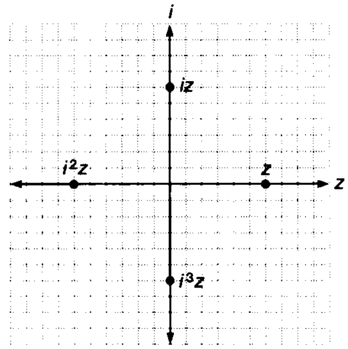


17. $2 + 6i$ **19.** $-9 - 7i$ **21.** $9 - 4i$
23. $-3 + 192i$ **25.** $-20 - 30i$
27. $-46 - 9i$ **29.** $\frac{33}{29} + \frac{10}{29}i$
31. $\frac{-12}{5} - \frac{9}{5}i$ **33.** 5.4 cis (−21.8°)
35. 3.2 cis 108.4° **37.** 5 cis 126.9°
39. 2 cis 30° **41.** $3\sqrt{2}$ cis 45°
43. $\sqrt{2}$ cis 225° **45.** 5 cis 90°
47. $2.9 + 0.8i$ **49.** $3.7 + 2.6i$
51. $1 - i$ **53.** $12.3 - 5.7i$
55. $\frac{3}{2} + \frac{\sqrt{3}}{2}i$ **57.** $5 - 5\sqrt{3}i$
59. $-\sqrt{10}$ **61.** $2 - 2i$ **63.** 15 cis 75°
65. 10.8 cis(−120°) **67.** 4 cis 80°
69. $\frac{20}{9}$ cis(−80°) **71.** 512 cis(−60°)
73. 27 cis(−120°) **75.** $-8.2 - 0.1i$
77. $2, -1 + \sqrt{3}i, -1 - \sqrt{3}i$
79. $3, 3i, -3, -3i$
81. $-1.1 + 4.9i, -3.7 - 3.4i, 4.8 - 1.5i$
83. 2.5 cis(−20°) **85.** 50 cis 45°
87. 2.04 cis 6.19° **89.** $0.75 + 0.86i$
91. Is the following an identity: a cis$(-\theta) = -a$ cis θ?
No. a cis$(-\theta)$
$= a\cos(-\theta) + a\sin(-\theta)i$
$= a\cos\theta - a\sin\theta i$
but $-a$ cis $\theta = -a\cos\theta - a\sin\theta i$.

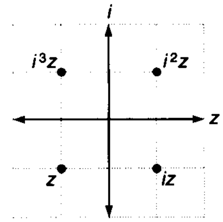
93. **a.** $-3 + i$ **b.** $-1 - 3i$
c. $3 - i$ **d.** $1 + 3i$



95. **a.** 6 **b.** $6i$ **c.** -6
d. $-6i$



97. **a.** $-1 - i$ **b.** $1 - i$
c. $1 + i$ **d.** $-1 + i$



99. **a.** $\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i$ **b.** $0.37 + 1.37i$
c. $0.37 - 1.37i$

101. $r^{\frac{1}{n}}\operatorname{cis}\left(\dfrac{\theta}{n} + \dfrac{k \cdot 360°}{n}\right)$

$$= r^{\frac{1}{n}}\operatorname{cis}\left[\frac{\theta}{n} + \frac{(an + b) \cdot 360°}{n}\right] \quad \text{Replace } k \text{ by } an + b.$$

$$\frac{\theta}{n} + \frac{(an + b) \cdot 360°}{n} = \frac{\theta}{n} + \frac{an \cdot 360°}{n} + \frac{b \cdot 360°}{n}$$

$$= \frac{\theta}{n} + a \cdot 360° + \frac{b \cdot 360°}{n}$$

$$= \left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°$$

$$= r^{\frac{1}{n}}\operatorname{cis}\left[\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°\right]$$

$$= r^{\frac{1}{n}}\cos\left[\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°\right] + ir^{\frac{1}{n}}\sin\left[\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°\right]$$

$$\cos\left[\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°\right] = \cos\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right)\cos(a \cdot 360°) - \sin\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right)\sin(a \cdot 360°)$$

$= \cos\left(\dfrac{\theta}{n} + \dfrac{b \cdot 360°}{n}\right)$, because $\cos(a \cdot 360°) = 1$ and $\sin(a \cdot 360°) = 0$, when a is an integer.

$$\sin\left[\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°\right] = \sin\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right)\cos(a \cdot 360°) + \cos\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right)\sin(a \cdot 360°)$$

$= \sin\left(\dfrac{\theta}{n} + \dfrac{b \cdot 360°}{n}\right)$, because $\cos(a \cdot 360°) = 1$ and $\sin(a \cdot 360°) = 0$,
when a is an integer.

Thus,

$$r^{\frac{1}{n}}\cos\left[\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°\right] + ir^{\frac{1}{n}}\sin\left[\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + a \cdot 360°\right]$$

$$= r^{\frac{1}{n}}\cos\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right) + ir^{\frac{1}{n}}\sin\left(\frac{\theta}{n} + \frac{b \cdot 360°}{n}\right)$$

$= r^{\frac{1}{n}}\operatorname{cis}\left(\dfrac{\theta}{n} + \dfrac{b \cdot 360°}{n}\right)$, where $b < n$.

This last expression is one of the previous roots.

Solutions to trial exercise problems

23. $(15 + 4i)(3 + 12i)$

$15(3) + 15(12i) + 4i(3) + 4i(12i)$

$45 + 180i + 12i + 48i^2 (i^2 = -1)$

$45 + 192i - 48$

$-3 + 192i$

27. $(2 - 3i)^3$

$(2 - 3i)(2 - 3i)(2 - 3i)$

$[(2 - 3i)(2 - 3i)](2 - 3i)$

$[4 - 12i + 9i^2](2 - 3i)$

$[-5 - 12i](2 - 3i)$

$-10 + 15i - 24i + 36i^2$

$-10 - 36 - 9i$

$-46 - 9i$

31. $\dfrac{3 + 6i}{-2 - i}$

$\dfrac{3 + 6i}{-2 - i} \cdot \dfrac{-2 + i}{-2 + i}$

Multiply by the conjugate of the denominator.

$\dfrac{-6 + 3i - 12i + 6i^2}{4 - 2i + 2i - i^2} = \dfrac{-6 - 9i - 6}{4 + 1}$

$\dfrac{-12 - 9i}{5} = -\dfrac{12}{5} - \dfrac{9}{5}i$

36. $\sqrt{3} - 2i$

$r = \sqrt{(\sqrt{3})^2 + (-2)^2} = \sqrt{7} \approx 2.6.$

$\theta' = \tan^{-1}\dfrac{-2}{\sqrt{3}} \approx -49.1°$; $a > 0$ to $\theta = \theta'$.

The point is $2.6 \text{ cis}(-49.1°)$.

39. $\sqrt{3} + i$

The modulus is $\sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2.$

$\tan\theta' = \dfrac{1}{\sqrt{3}}$, so $\theta' = 30°$ (exactly).

$\theta = \theta' = 30°$, since θ is in the first quadrant. Thus, the polar form is 2 cis 30°.

47. 3 cis 15°

$3 \cos 15° + (3 \sin 15°)i = 2.9 + 0.8i$

58. 6 cis 135°

$6(\cos 135° + i \sin 135°)$

$6\left(-\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i\right)$

$-3\sqrt{2} + 3\sqrt{2}i$

73. $(3 \text{ cis } 200°)^3 = 3^3 \text{ cis } 3(200)°$

$= 27 \text{ cis } 600°$

$= 27 \text{ cis}(600° - 2 \cdot 360°)$

$= 27 \text{ cis}(-120°)$

76. $(0.8 + 0.6i)^{10}$

We first put the complex number in polar form. The modulus is $\sqrt{0.8^2 + 0.6^2} = \sqrt{1} = 1.$

$\tan\theta' = \dfrac{0.6}{0.8} = \dfrac{3}{4}$, so $\theta' = 36.8699°$.

Since θ terminates in quadrant I, $\theta = 36.8699°$. We now compute:

$(1 \text{ cis } 36.8699)^{10} = 1^{10} \text{ cis } 10(36.8699)° = 1 \text{ cis } 368.699° = 1 \text{ cis } 8.699°$. Since the original complex number was in its rectangular form, we will put this result in rectangular form.

$1 \text{ cis } 8.699° = \cos 8.699° + i \sin 8.699°$

$= 0.99 + 0.15i = 1.0 + 0.2i$

82. Find the 4 fourth roots of $\sqrt{3} + 3i$ to the nearest tenth.

$\sqrt{3} + 3i = 2\sqrt{3} \text{ cis } 60°$, so

$(\sqrt{3} + 3i)^{1/4} = (2\sqrt{3} \text{ cis } 60°)^{1/4}$

Evaluate $(2\sqrt{3})^{1/4}\text{cis}\left(\dfrac{60°}{4} + \dfrac{k \cdot 360°}{4}\right)$

$\approx 1.364 \text{ cis}(15° + k \cdot 90°)$ for $k = 0, 1, 2, 3.$

$k = 0$: $1.364 \text{ cis}(15°)$

$= 1.364 \cos 15° + 1.364 \sin 15° i$

$\approx 1.3 + 0.4i$

$k = 1$: $1.364 \text{ cis}(15° + 90°)$

$= 1.364 \cos 105° + 1.364 \sin 105° i$

$\approx -0.4 + 1.3i$

$k = 2$: $1.364 \text{ cis}(15° + 180°)$

$= 1.364 \cos 195° + 1.364 \sin 195° i$

$\approx -1.3 - 0.4i$

$k = 3$: $1.364 \text{ cis}(15° + 270°)$

$= 1.364 \cos 285° + 1.364 \sin 285° i$

$\approx 0.4 - 1.3i$

87. $Z_T = \dfrac{Z_1 Z_2}{Z_1 + Z_2} = \dfrac{(2 + i)(3 - 5i)}{(2 + i) + (3 - 5i)}$

$= \dfrac{(2 + i)(3 - 5i)}{5 - 4i} = \dfrac{6 - 7i + 5}{5 - 4i}$

$= \dfrac{11 - 7i}{5 - 4i} = \dfrac{13.038 \text{ cis } 327.529°}{6.403 \text{ cis } 321.340°}$

$= \dfrac{13.038}{6.403} \text{cis}(327.529° - 321.340°)$

$= 2.04 \text{ cis } 6.19°$

93. **a.** $-3 + i$

b. $i(-3 + i) = -3i + i^2$

$= -3i - 1$

$= -1 - 3i$

c. $i^2(-3 + i) = -1(-3 + i)$

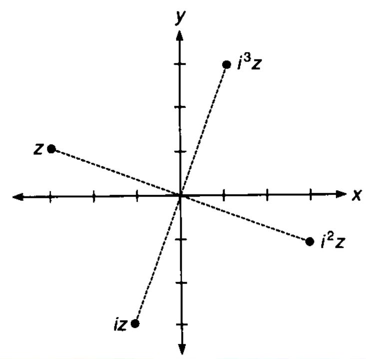
$= 3 - i$

d. $i^3 z = i i^2 z = i(-1)z$

$= -iz = -i(-3 + i)$

$= 3i - i^2 = 3i - (-1)$

$= 1 + 3i$



100. $\dfrac{r_1\text{cis }\theta_1}{r_2\text{cis }\theta_2} = \dfrac{r_1\cos\theta_1 + ir_1\sin\theta_1}{r_2\cos\theta_2 + ir_2\sin\theta_2}$

$= \dfrac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)}$

$= \dfrac{r_1}{r_2} \cdot \dfrac{\cos\theta_1 + i\sin\theta_1}{\cos\theta_2 + i\sin\theta_2}$

$= \dfrac{r_1}{r_2} \cdot \dfrac{\cos\theta_1 + i\sin\theta_1}{\cos\theta_2 + i\sin\theta_2} \cdot \dfrac{\cos\theta_2 - i\sin\theta_2}{\cos\theta_2 - i\sin\theta_2}$

$= \dfrac{r_1}{r_2} \cdot \dfrac{\cos\theta_1\cos\theta_2 - i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2 - i^2\sin\theta_1\sin\theta_2}{\cos^2\theta_2 - i^2\sin^2\theta_2}$

$= \dfrac{r_1}{r_2} \cdot \dfrac{\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 + i(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)}{\cos^2\theta_2 + \sin^2\theta_2}$

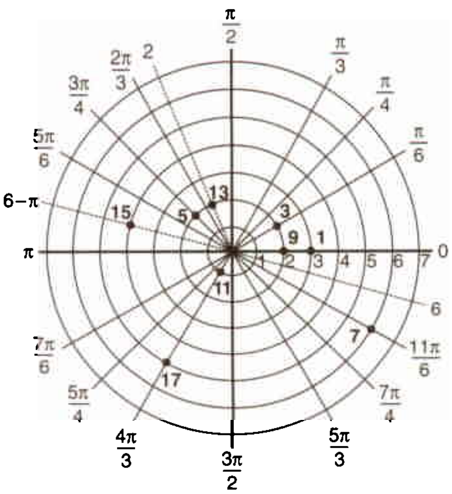
$= \dfrac{r_1}{r_2} \cdot \dfrac{\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)}{1}$

$= \dfrac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

Exercise 7–2

Answers to odd-numbered problems

The figure shows the answers to odd-numbered problems 1 through 17.



19. $\left(2,\frac{13\pi}{6}\right)\left(2,\frac{25\pi}{6}\right)\left(-2,\frac{7\pi}{6}\right)$

21. $\left(6,\frac{23\pi}{6}\right)\left(6,-\frac{\pi}{6}\right)\left(-6,\frac{5\pi}{6}\right)$

23. $(2,2+2\pi)\ (2,2+4\pi)\ (-2,2+\pi)$

25. $(0,4)$ **27.** $\left(\frac{-5\sqrt{3}}{2},\frac{5}{2}\right)$

29. $(-2,-2\sqrt{3})$ **31.** $(1.08,1.68)$

33. $(2.05,2.19)$ **35.** $(-2.61,-3.03)$

37. $\left(4,-\frac{5\pi}{6}\right)$ **39.** $(2,\pi)$

41. $\left(4\sqrt{2},-\frac{3\pi}{4}\right)$ **43.** $(3.61,0.98)$

45. $(4.12,-1.33)$ **47.** $(6.40,0.67)$

49. $\tan\theta = 4$ **51.** $r = \dfrac{2}{\sin\theta + 3\cos\theta}$

53. $r = \dfrac{b}{\sin\theta - m\cos\theta}$

55. $r^2 = \dfrac{5}{\sin^2\theta - 2\cos^2\theta}$

57. $r^2 = \dfrac{1}{\cos^2\theta + 2}$ **59.** $x^2 + y^2 = y$

61. $x = 2$ **63.** $(x^2 + y^2)^3 = 36x^2y^2$

65. $x^4 + 2x^2y^2 + y^4 = 2xy$

67. $x^3 + xy^2 = y$

69. $x^2 + y^2 = 4y^2 + 12y + 9$

71. $2(r\cos\theta\ r\sin\theta) = 5$, $r^2\sin 2\theta = 5$,

$r^2 = \dfrac{5}{\sin 2\theta} = 5\csc 2\theta$

73. Consider a point $P = (r,\theta)$, where $r < 0$. Then $P = (-r,\theta + \pi)$, where $-r > 0$. Therefore, since $-r > 0$, $y = -r\sin(\theta + \pi)$ is true,

$y = -r\sin(\theta + \pi)$

A true statement.

$= -r(\sin\theta\cos\pi + \cos\theta\sin\pi)$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= -r(\sin\theta(-1) + \cos\theta(0))$

$= -r(-\sin\theta)$

$= r\sin\theta$

Thus, $y = r\sin\theta$, even if $r < 0$.

75. $(x^2 + y^2 + 2y)^2 = x^2 + y^2$

77. $(x^2 + y^2)^3 = (x^2 + y^2 + 4xy)^2$

79. $(x^2 + y^2 + 3x)^2 = x^2 + y^2$

Solutions to trial exercise problems

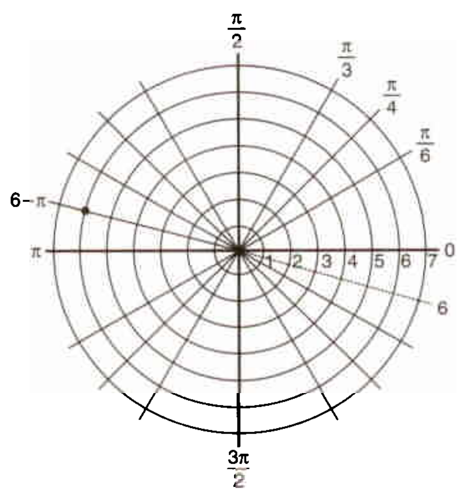
15. $(-4,6)$ represents a point with $r = -4$ and $\theta = 6$ (radians). We can convert r to a positive value if we add or subtract π from θ. Since $6 > \pi$, subtract:

$(-4,6) = (4,6 - \pi)$

$= (4,2.9)$ (approximately).

(See the diagram.)

To construct an angle of 2.9 radians convert it to $\approx 166°$.



21. $\left(6,\frac{11\pi}{6}\right)$

Add 2π to θ:

$\left(6,\frac{11\pi}{6}\right) = \left(6,\frac{11\pi}{6} + \frac{12\pi}{6}\right)$

$= \left(6,\frac{23\pi}{6}\right)$

Subtract 2π from θ:

$\left(6,\frac{11\pi}{6}\right) = \left(6,\frac{11\pi}{6} - \frac{12\pi}{6}\right)$

$= \left(6,-\frac{\pi}{6}\right)$

To obtain $r < 0$, add or subtract an odd multiple of π. We subtract π.

$\left(6,\frac{11\pi}{6}\right) = \left(-6,\frac{11\pi}{6} - \frac{6\pi}{6}\right)$

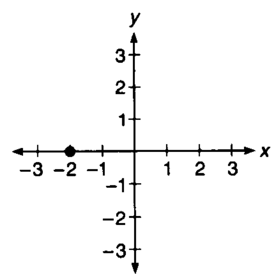
$= \left(-6,\frac{5\pi}{6}\right)$

31. $(2,1)$

$(2,1) = (2\cos 1, 2\sin 1)$

$= (1.08,1.68)$

39. $(-2,0)$ is plotted in the diagram. We can see that $\theta = 180°$ or π (radians), and that $r = 2$. Thus, $(-2,0)$ (rectangular) $= (2,\pi)$ (polar).



45. $(1,-4)$

$r = \sqrt{1^2 + 4^2} = \sqrt{17} \approx 4.12$

$\theta' = \tan^{-1}(-4) \approx -1.326$

$x > 0$ so $\theta = \theta'$.

$(4.12,-1.33)$

53. $y = mx + b,\ b \neq 0$
Use $x = r\cos\theta$, $y = r\sin\theta$.
$y = mx + b$
$r\sin\theta = m(r\cos\theta) + b$
Solve for r when possible.
$r\sin\theta - mr\cos\theta = b$
$r(\sin\theta - m\cos\theta) = b$
$$r = \frac{b}{\sin\theta - m\cos\theta}$$
The quotient is defined when
$\sin\theta - m\cos\theta \neq 0$
$\sin\theta \neq m\cos\theta$
$\frac{\sin\theta}{\cos\theta} \neq m$, $\cos\theta \neq 0$
$\tan\theta \neq m$, $\cos\theta \neq 0$
$\cos\theta = 0$ implies that θ is an odd multiple of $\frac{\pi}{2}$, representing a vertical line. However, $y = mx + b$ can only represent a nonvertical line, so the condition $\cos\theta \neq 0$ is satisfied.
If $\tan\theta = m$, since $\tan\theta = \frac{y}{x}$ if θ is not an odd multiple of $\frac{\pi}{2}$, we have $\frac{y}{x} = m$, or $y = mx$. This would be a line parallel to $y = mx + b$ with $b = 0$, but the problem specifies that $b \neq 0$. Thus, we are also guaranteed that $\tan\theta \neq m$. Therefore, under all conditions for which $y = mx + b$, $b \neq 0$ are satisfied, the polar equation is $r = \frac{b}{\sin\theta - m\cos\theta}$.

55. $y^2 - 2x^2 = 5$
$(r\sin\theta)^2 - 2(r\cos\theta)^2 = 5$
$r^2\sin^2\theta - 2r^2\cos^2\theta = 5$
$r^2(\sin^2\theta - 2\cos^2\theta) = 5$
$$r^2 = \frac{5}{\sin^2\theta - 2\cos^2\theta}$$

63. $r = 3\sin 2\theta$
$r = 3\sin 2\theta$
$\sin 2\theta = 2\sin\theta\cos\theta$
$= 3(2\sin\theta\cos\theta)$
$= 6\frac{y}{r}\cdot\frac{x}{r}$
$r^3 = 6xy$
$r = \sqrt{x^2 + y^2} = (x^2 + y^2)^{1/2}$
$[(x^2 + y^2)^{1/2}]^3 = 6xy$
$(x^2 + y^2)^{3/2} = 6xy$, or $(x^2 + y^2)^3 = 36x^2y^2$.

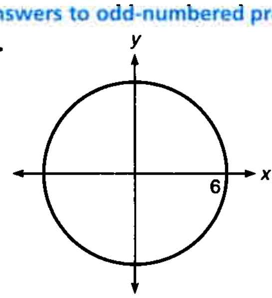
69. $r = \frac{3}{1 - 2\sin\theta}$ (Note $r \neq 0$ because the quotient cannot be 0.)
$$r = \frac{3}{1 - 2\frac{y}{r}}$$
$$= \frac{3}{1 - 2\frac{y}{r}}\cdot\frac{r}{r} = \frac{3r}{r - 2y}$$
$r(r - 2y) = 3r$; $r^2 - 2ry = 3r$;
$r^2 = 3r + 2ry$
$r^2 = r(3 + 2y)$
Divide by r, since $r \neq 0$.
$r = 3 + 2y$
$(x^2 + y^2)^{1/2} = 3 + 2y$
$x^2 + y^2 = (3 + 2y)^2$
$x^2 + y^2 = 4y^2 + 12y + 9$

78. Problem 79 in section 5–3 shows that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.
$r = 2\cos 3\theta$
$r = 2(4\cos^3\theta - 3\cos\theta)$
$r = 8\cos^3\theta - 6\cos\theta$
$$r = 8\left(\frac{x}{r}\right)^3 - 6\frac{x}{r}$$
$$r = \frac{8x^3}{r^3} - \frac{6x}{r}$$
$r^4 = 8x^3 - 6xr^2$
Multiply each member by r^3.
$(x^2 + y^2)^2 = 8x^3 - 6x(x^2 + y^2)$
$x^4 + 2x^2y^2 + y^4 = 8x^3 - 6x^3 - 6xy^2$
$x^4 - 2x^3 + 2x^2y^2 + 6xy^2 + y^4 = 0$

Exercise 7–3

Answers to odd-numbered problems

1.



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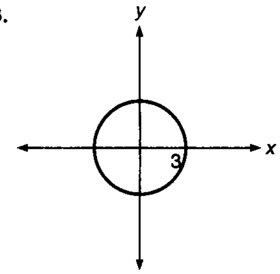

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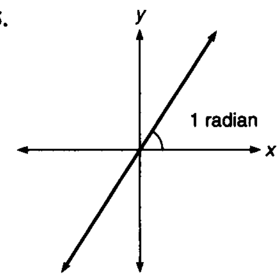

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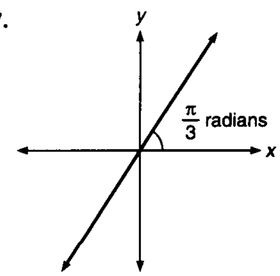

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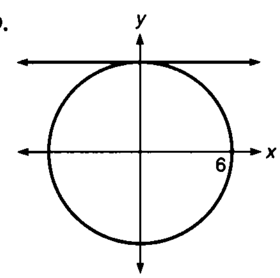

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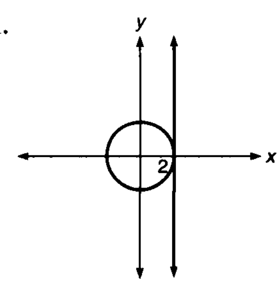

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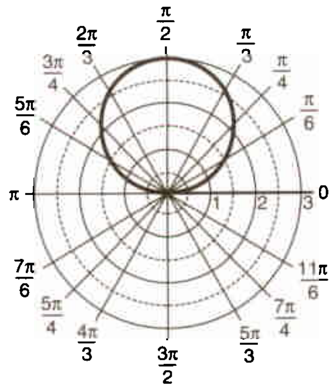

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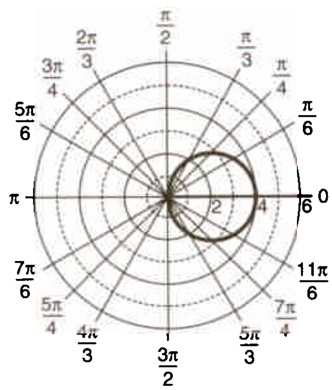

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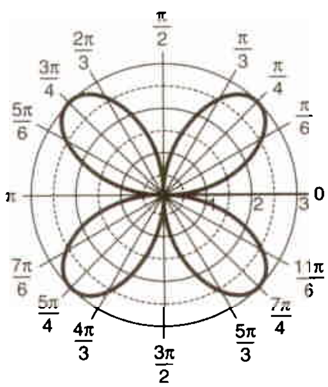

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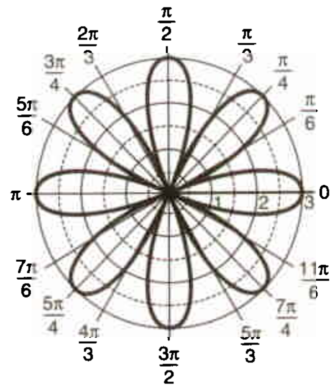

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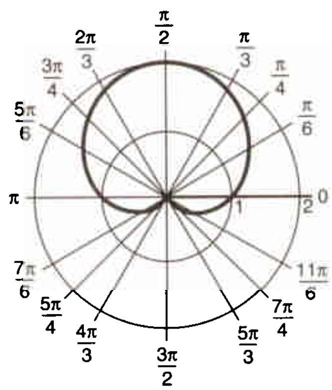

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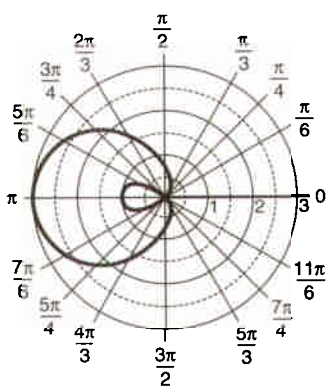

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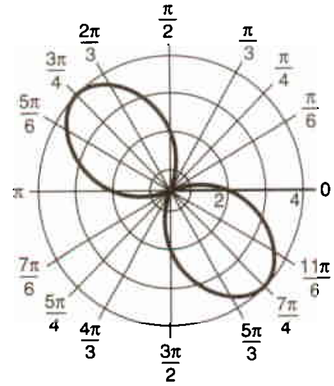

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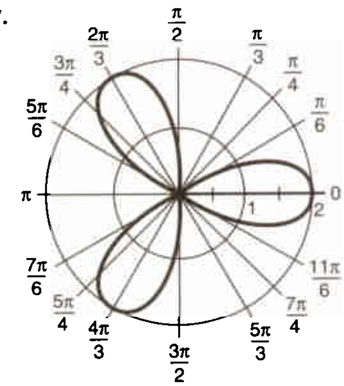

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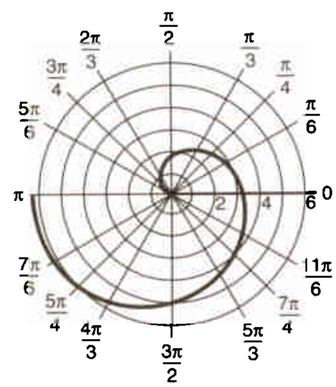

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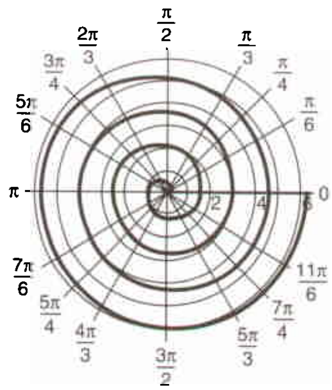

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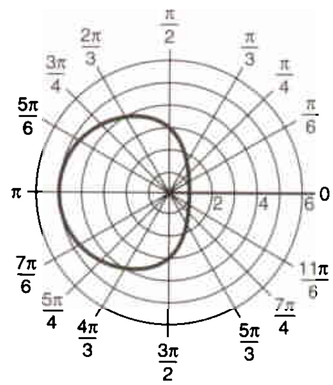

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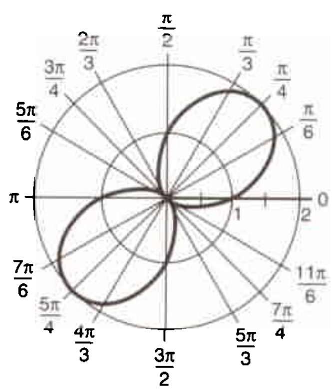

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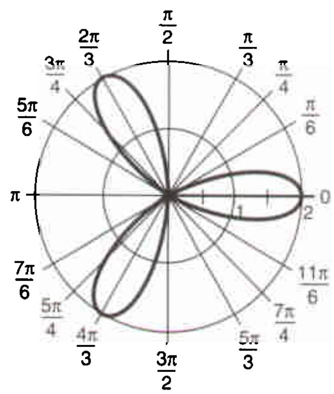
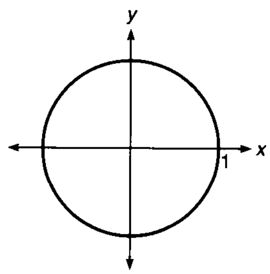


Solutions to trial exercise problems

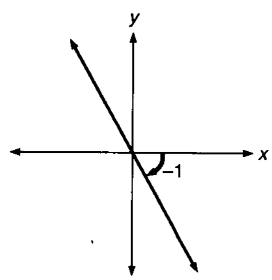
4. $r = -1$

Since any point $(-1,\theta)$ is equivalent to the point $(1,\theta + \pi)$, and since θ can be any value (since it is not specified), the graph of this equation is equivalent to the graph of $r = 1$, which is a circle with radius 1, centered at the pole.



8. $\theta = -1$

The radius r can be any value, but the angle must be -1 (radians). This produces points at all distances from the pole, along the line in which the angle is -1 (radians) or about $-57°$.

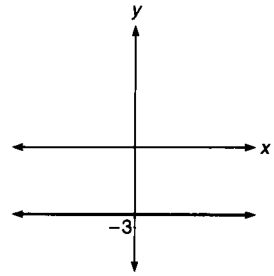


12. $r \sin \theta = -3$

$y = -3$

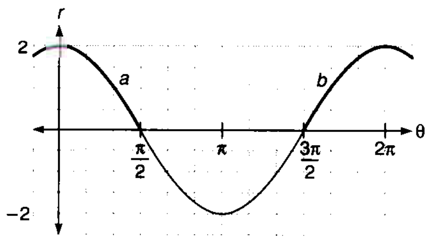
Replace $r \sin \theta$ by y.

This is the horizontal line $y = -3$.

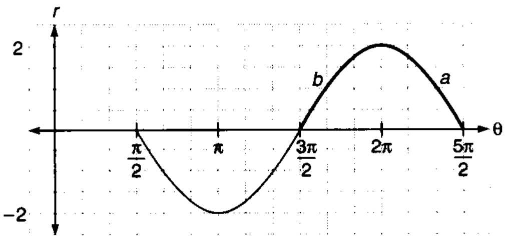


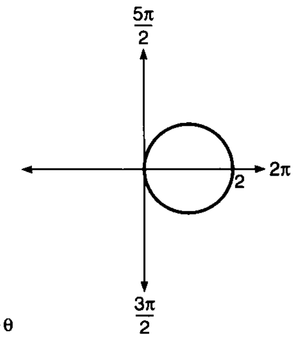
14. $r = 2 \cos \theta$

The following graph shows the rectangular coordinate graph of $r = 2 \cos \theta$. If we move the "negative lobe" between $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ by adding π to all x values, it becomes a positive lobe between $\frac{3\pi}{2}$ and $\frac{5\pi}{2}$. This just repeats the positive lobe that starts at $\frac{3\pi}{2}$ in the graph. Thus, we can ignore the negative lobe for finding the polar graph.



It is easier to imagine what is happening if we move the lobe that starts at 0 to start at 2π. We then see that we have a lobe between $\frac{3\pi}{2}$ and $\frac{5\pi}{2}$, with its maximum value at 2π.

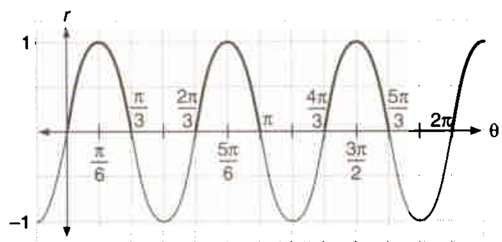


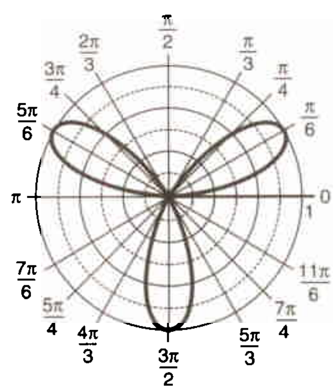


This positive lobe produces a circle between $\frac{3\pi}{2}$ and $\frac{5\pi}{2}$. It takes some experience or a lot of point plotting to know that this lobe is a circle. Remember, however, that graphs of the form $r = k \sin \theta$ or $r \approx k \cos \theta$ produce circles.

20. $r = \sin 3\theta$

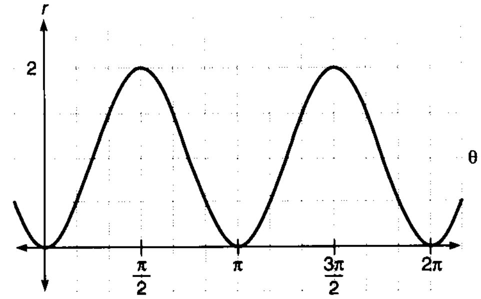
The rectangular coordinate graph of $r = \sin 3\theta$ is shown below. The negative lobes can be ignored because shifting them by π units and flipping them about the θ axis causes them to overlap the positive lobes. We see there are lobes between 0 and $\frac{\pi}{3}$, with maximum at $\frac{\pi}{6}$, $\frac{2\pi}{3}$, and π, with maximum at $\frac{5\pi}{6}$, and $\frac{4\pi}{3}$ and $\frac{5\pi}{3}$, with maximum at $\frac{3\pi}{2}$.

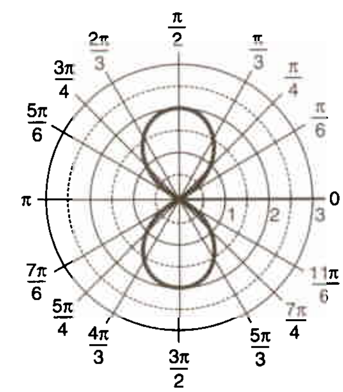




These lobes produce the polar graph shown.

26. $r = 1 - \cos 2\theta$



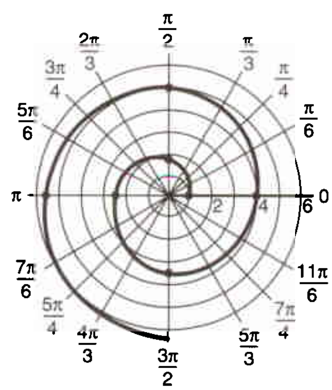


The rectangular graph shows two lobes, one between 0 and π, with maximum at $\frac{\pi}{2}$, and one between π and 2π, with maximum at $\frac{3\pi}{2}$. The two positive lobes produce the polar graph shown. The two lobes are not circles, which would require algebra beyond the scope of this text to show. Of the problems in this text, only those equations categorized in the text as producing circles will in fact produce circles.

32. $r = 1 + \frac{\theta}{2}$

This graph is not periodic, and thus an analysis of its rectangular graph is less helpful than in cases where the trigonometric functions are involved. We simply plot points to obtain the graph. It is clear that as the value of θ increases, r increases. This property produces a spiral, which starts at $r = 1$. A table of values and the graph are shown.

θ	r
0	1
$\frac{\pi}{2}$	1.8
π	2.6
$\frac{3\pi}{2}$	3.4
2π	4.1
$\frac{5\pi}{2}$	4.9
3π	5.7

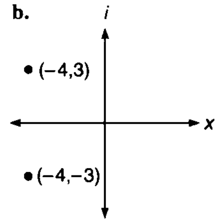


Chapter 7 review

1. real, 3; imaginary, −1 **2.** real, 0; imaginary, 3 **3.** real, 12; imaginary, 0 **4.** real, 0; imaginary, −1 **5.** $-1 - 2i$ **6.** $6 + i$ **7.** $63 + 48i$ **8.** $24 + 27i$ **9.** $12 + 8i$ **10.** $53 - 60i$ **11.** $\frac{42}{29} - \frac{40}{29}i$ **12.** $\frac{-2}{5} - \frac{3}{10}i$ **13.** $\frac{3}{7} - \frac{4}{7}i$ **14.** $1 + 4i$ **15.** $-2 + 4i$ **16.** $\frac{-4}{5} - \frac{7}{5}i$ **17.** $\frac{-25}{26} + \frac{5}{26}i$

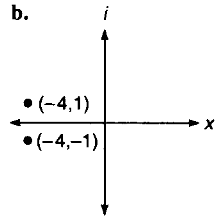
18. a. $-4 - 3i$

b.



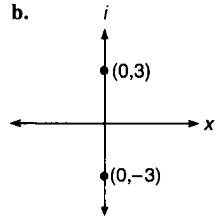
19. a. $-4 - i$

b.



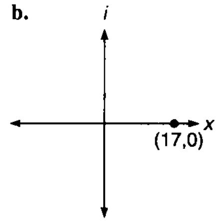
20. a. $-3i$

b.



21. a. 17

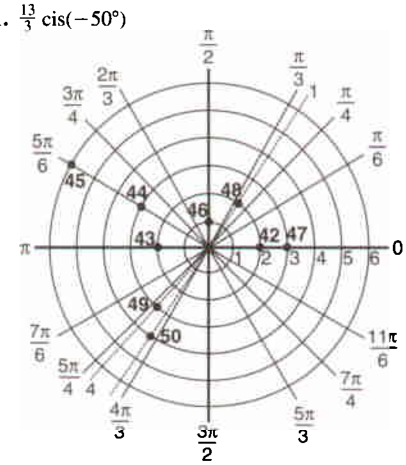
b.



22. 3.6 cis(−33.7°) **23.** 3.5 cis 60° **24.** 2.2 cis(−116.6°) **25.** 2 cis(−30°) **26.** $3\sqrt{2}$ cis 45° **27.** 4 cis(−90°) **28.** $2.5 + 1.7i$ **29.** $-1.4 - 2.7i$ **30.** $-\frac{3}{2} - \frac{3\sqrt{3}}{2}i$ **31.** $5\sqrt{3} - 5i$ **32.** 6 cis 70° **33.** 13 cis 140° **34.** 8 cis 100° **35.** $\frac{1}{2}$ cis 36° **36.** 8 cis 30° **37.** 16 cis(−120°) **38.** $0.4 - 0.9i$ **39.** $2, 2i, -2, -2i$

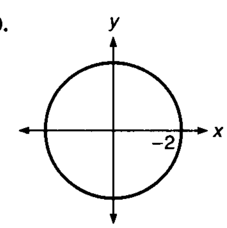
40. $\frac{3}{2} + \frac{3\sqrt{3}}{2}i, -3, \frac{3}{2} - \frac{3\sqrt{3}}{2}i$

41. $\frac{13}{3}$ cis(−50°)

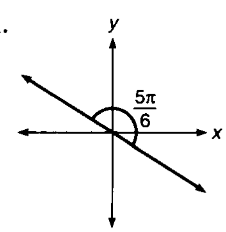


51. $\left(\frac{3\sqrt{3}}{2}, \frac{-3}{2}\right)$ **52.** $(-2, 2\sqrt{3})$ **53.** $(-1, \sqrt{3})$ **54.** (1.08,1.68) **55.** (−2.08,4.55) **56.** (0.88,0.48) **57.** (2.24,0.46) **58.** (5.83,2.60) **59.** (4.12,−1.82) **60.** $\tan\theta = -3$ **61.** $r = \frac{2}{\sin\theta - 4\cos\theta}$ **62.** $r^2 = \frac{5}{2 - 3\cos^2\theta}$ **63.** $r = \frac{3\cos\theta}{\sin^2\theta}$ **64.** $x^2 + y^2 = y$ **65.** $x = 2$ **66.** $x^4 + 2x^2y^2 + y^4 = 2xy$ **67.** $x^3 + xy^2 = y$ **68.** $y = 2$ **69.** $4x^2 + 3y^2 - 6y - 9 = 0$

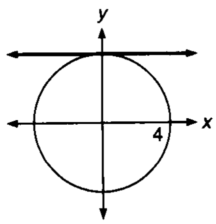
70.



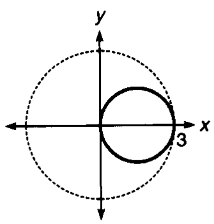
71.



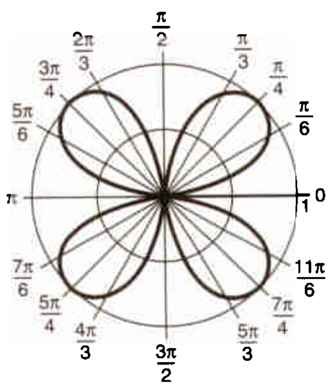
72. $2r \sin \theta = 8$
$r \sin \theta = 4$
$y = 4$



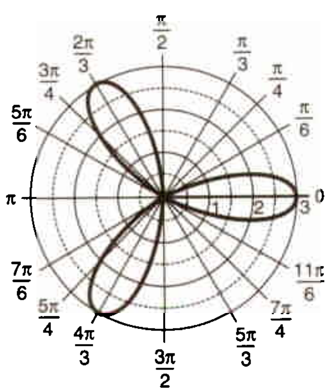
73. $r = 3 \cos \theta$



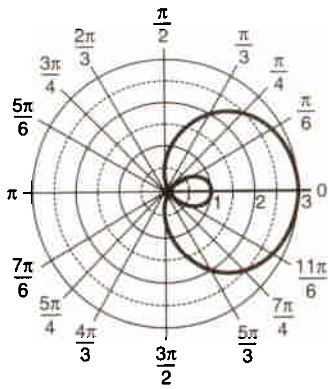
74.



75.



76.



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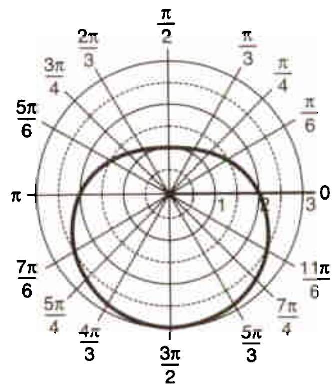
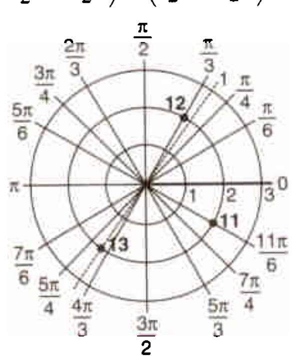


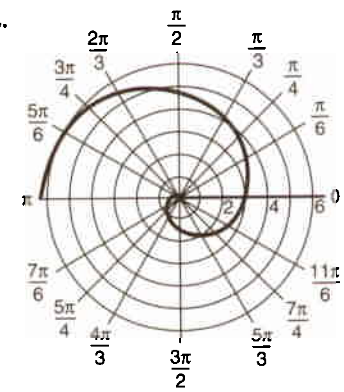
Chapter 7 test

1. $15 - 6i$ **2.** 34 **3.** $\frac{19}{13} - \frac{4}{13}i$
4. $-6 + 8i$ **5.** $6.4 \text{ cis}(-51.3°)$
6. $-1 + \sqrt{3}i$ **7.** $14 \text{ cis } 110°$
8. $3 \text{ cis } 120°$ **9.** $27 \text{ cis } 90°$
10. $2\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right), 2\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right),$
$2\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right), 2\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$

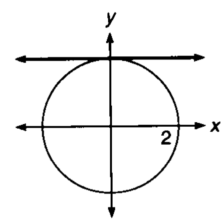


14. (2.09,2.15) **15.** (2.08,−4.55)
16. $\left(2, -\frac{5\pi}{6}\right)$ **17.** $\left(5\sqrt{2}, \frac{3\pi}{4}\right)$
18. $r = \dfrac{5}{\sin \theta + 3 \cos \theta}$
19. $2r^2\sin^2\theta - r \cos \theta = 5$
20. $y = 2$
21. $x^4 + 2x^2y^2 + y^4 = x^2 - y^2$

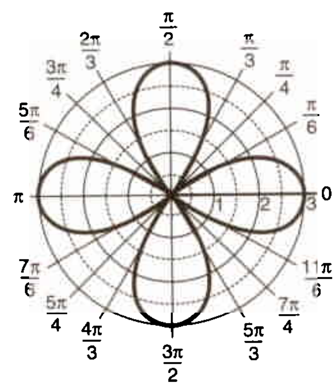
22.



23. $r = 2 \csc \theta$
$r = \dfrac{2}{\sin \theta}$
$r \sin \theta = 2$
$y = 2$



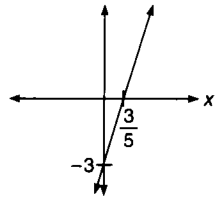
24.



Appendix A

Answers to odd-numbered problems

1.



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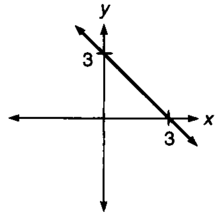

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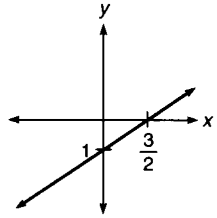

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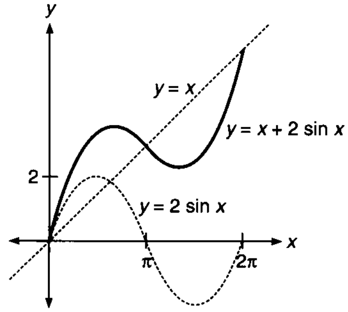

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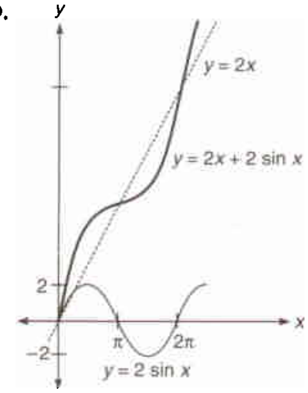
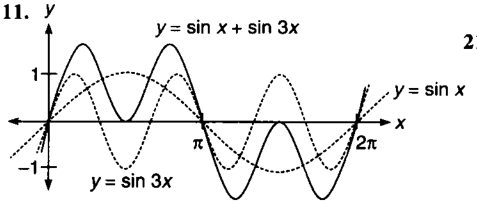
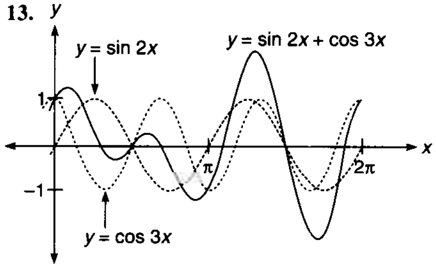


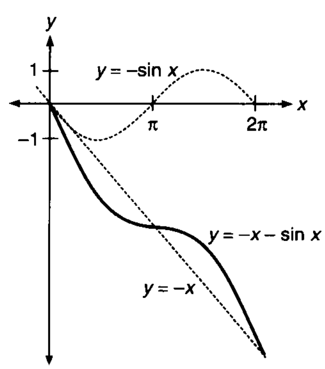
11.



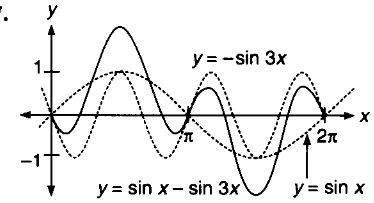
13.



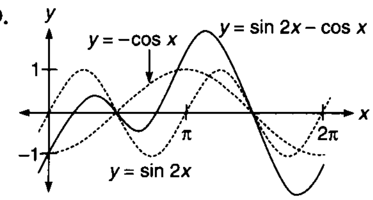
15.



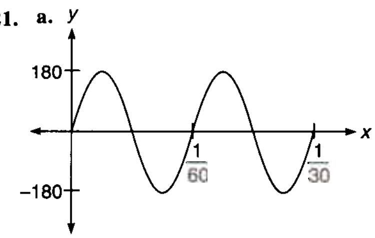
17.



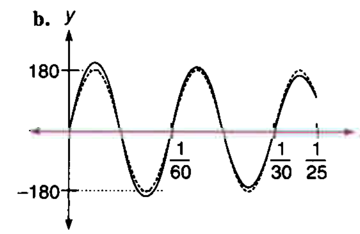
19.



21. a.



b.



Solutions to trial exercise problems

5. $f(x) = \frac{2}{3}x - 1$; $y = \frac{2}{3}x - 1$

Let $x = 0$: $y = -1$, so $(0, -1)$ is the y-intercept.

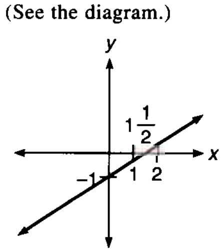
Let $y = 0$:

$0 = \frac{2}{3}x - 1$

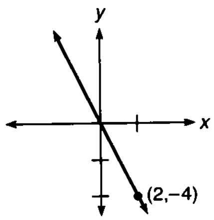
$1 = \frac{2}{3}x$ Multiply by $\frac{3}{2}$.

$\frac{3}{2} = x$, so $(\frac{3}{2}, 0)$ is the x-intercept.

(See the diagram.)



6. $f(x) = -2x$; $y = -2x$
By letting $x = 0$ and then $y = 0$, we find that the x- and y-intercepts are the same point (0,0). Thus, to graph this function we need another point. To get another point, we can let x (or y) be any value but 0, which we have already used. Let $x = 2$; thus, $y = -4$, and $(2,-4)$ is another point to plot. (See the diagram.)



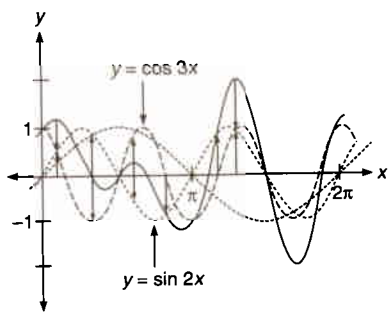
13. $y = \sin 2x + \cos 3x$
First we graph $y = \sin 2x$:

$$0 \le 2x \le 2\pi$$
$$0 \le x \le \pi$$

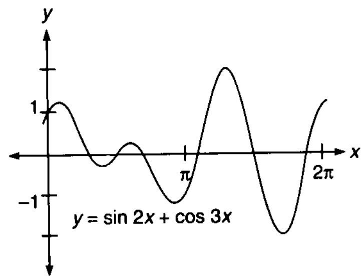
(shown in short dashed lines in the diagram)
and $y = \cos 3x$:

$$0 \le 3x \le 2\pi$$
$$0 \le x \le \frac{2\pi}{3}$$

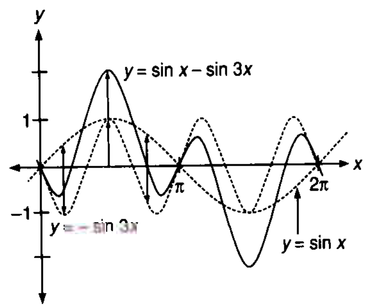
(shown in long dashed lines).



Next we add the ordinates. Remember that wherever one function is zero (crosses the x-axis), the result is the other function. Wherever the functions have equal absolute values but opposite signs, the result is zero. Wherever both functions cross, the result is twice the height of either function. (See the diagram.)



17. $y = \sin x - \sin 3x$
We rewrite the function as $y = \sin x + (-\sin 3x)$. We now graph $y = \sin x$ and $y = -\sin 3x$, and add the ordinates. (See the diagram.)



20. $y = 7.5 \sin \frac{\pi}{6}x + 10 \sin \frac{\pi}{3}x$

$0 \le \frac{\pi}{6}x \le 2\pi$. Multiply by $\frac{6}{\pi}$.

$\frac{6}{\pi} \cdot 0 \le \frac{6}{\pi} \cdot \frac{\pi}{6}x \le \frac{6}{\pi} \cdot 2\pi$; $0 \le x \le 12$.

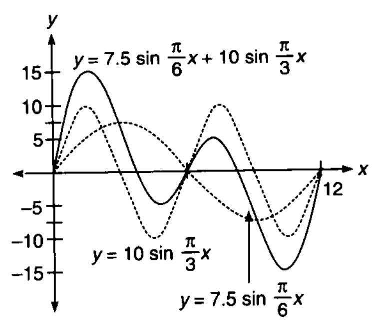
Similarly,

$0 \le \frac{\pi}{3}x \le 2\pi$. Multiply by $\frac{3}{\pi}$.

$\frac{3}{\pi} \cdot 0 \le \frac{3}{\pi} \cdot \frac{\pi}{3}x \le \frac{3}{\pi} \cdot 2\pi$;

$0 \le 2x \le 12$; $0 \le x \le 6$.

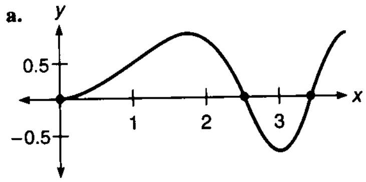
(See the diagram.)



Appendix B

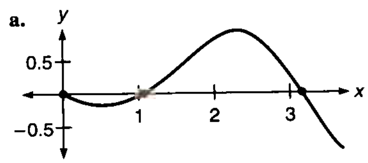
Answers to odd-numbered problems

1. a.



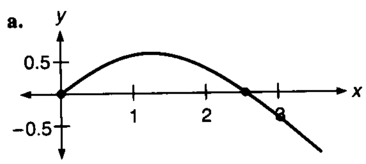
b. 0, 2.5, 3.5 c. 0, 2.50662827, 3.54490770

3. a.



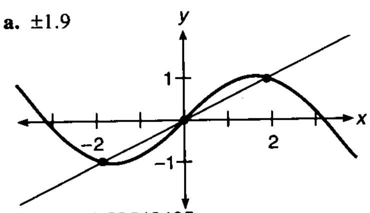
b. 0, 1.1, 3.2 c. 0, 1.04719755, 3.14159265 (π)

5. a.



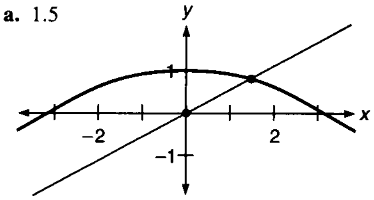
b. 0, 2.5 c. 0, 2.47457679

7. a. ± 1.9



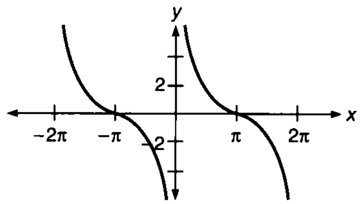
b. 0, $\pm$ 1.89549427

9. a. 1.5

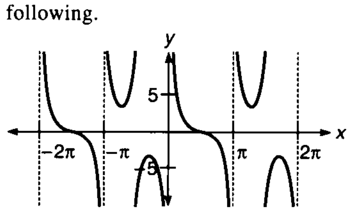


b. 1.47817027

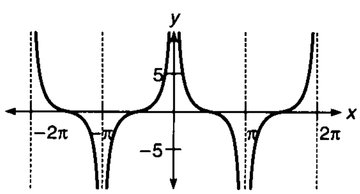
11. The graph of $y = \csc\theta + \cot\theta$, and $y = \dfrac{1+\cos\theta}{\sin\theta}$ both look like the following.



13. The graphs of $y = \dfrac{\csc\theta}{\sec\theta + \tan\theta}$ and of $y = \dfrac{\cos\theta}{\sin\theta + \sin^2\theta}$ both look like the following.



15. The graph of both $y = \dfrac{1}{\sec\theta - \cos\theta}$ and of $y = \cot\theta\csc\theta$ are as follows.

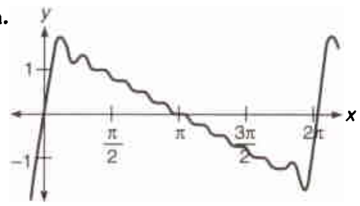


17. See the answers to appendix A.

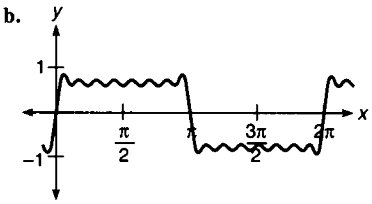
19. See the answers to appendix A, problem 21.

21. Each graph done with the first 12 terms.

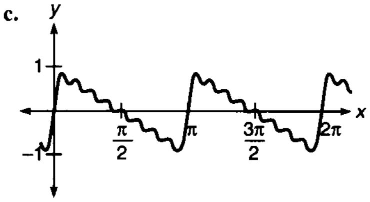
a.



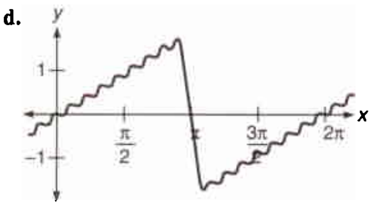
b.



c.



d.



Solutions to trial exercise problems

22. a.

x	$\sin(x)$	$\sin[(4\pi + 1)x]$	$\sin[(8\pi + 1)x]$	$\sin[(12\pi + 1)x]$
0	0.00	0.00	0.00	0.00
0.5	0.48	0.48	0.48	0.48
1	0.84	0.84	0.84	0.84
1.5	1.00	1.00	1.00	1.00
2	0.91	0.91	0.91	0.91
2.5	0.60	0.60	0.60	0.60
3	0.14	0.14	0.14	0.14
3.5	−0.35	−0.35	−0.35	−0.35
4	−0.76	−0.76	−0.76	−0.76
4.5	−0.98	−0.98	−0.98	−0.98
5	−0.96	−0.96	−0.96	−0.96
5.5	−0.71	−0.71	−0.71	−0.71
6	−0.28	−0.28	−0.28	−0.28
6.5	0.22	0.22	0.22	0.22

All functions of the form $y = \sin[(4k\pi + 1)x]$ are equal if x is an integer multiple of $\frac{1}{2}$.

b. The second column of part b of the next table shows the values that these functions produce. They are the same for the four functions. All functions of the form $y = \sin[(8k\pi + 1)x]$ are equal if x is an integer multiple of $\frac{1}{4}$.

c. The second column of part c of the table below shows the values that these functions produce. They are the same for the four functions.
All functions of the form $y = \sin[(20k\pi + 1)x]$ are equal if x is an integer multiple of $\frac{1}{10}$.

Part b

x	$\sin[(8\pi + 1)x]$
0	0.00
0.25	0.25
0.5	0.48
0.75	0.68
1	0.84
1.25	0.95
1.5	1.00
1.75	0.98
2	0.91
2.25	0.78
2.5	0.60
2.75	0.38
3	0.14
3.25	−0.11

Part c

x	$\sin[(20k\pi + 1)x]$
0	0.00
0.1	0.10
0.2	0.20
0.3	0.30
0.4	0.39
0.5	0.48
0.6	0.56
0.7	0.64
0.8	0.72
0.9	0.78
1	0.84
1.1	0.89
1.2	0.93
1.3	0.96

d. In parts a, b, c, we notice that $\frac{1}{2} \cdot 4k\pi = 2k\pi$, $\frac{1}{4} \cdot 8k\pi = 2k\pi$, $\frac{1}{10} \cdot 20k\pi = 2k\pi$, so we know that for $\frac{1}{100}$ we need $\frac{1}{100} \cdot \beta k\pi = 2k\pi$, so $\beta = 200$. Thus, all functions of the form $y = \sin[(200k\pi + 1)x]$ are equal if x is an integer multiple of $\frac{1}{100}$.

e. To see why the generalization of part a is true we consider the following:
If x is an integer multiple of $\frac{1}{2}$ we can describe it as $x = \dfrac{n}{2}$ for some integer n. Then,

$$\begin{aligned}\sin[(4k\pi + 1)x] &= \sin\left[\left(4k\pi + 1\right) \cdot \frac{n}{2}\right] = \sin\left(2nk\pi + \frac{n}{2}\right)\\ &= \sin 2nk\pi \cos\frac{n}{2} + \cos 2nk\pi \sin\frac{n}{2}\\ &= 0 \cdot \cos\frac{n}{2} + 1 \cdot \sin\frac{n}{2}\\ &= \sin\frac{n}{2} = \sin x\end{aligned}$$

Thus, as long as x is an integer multiple of $\frac{1}{2}$, $\sin[(4k\pi + 1)x] = \sin x$.

Similar reasoning shows why the generalizations of parts b, c, and d are true.

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